Binary and Decimal Numbers

What is a binary number?

- A binary number is a number that includes only ones and zeroes.
- The number could be of any length
- The following are all examples of binary numbers

0	10101
1	0101010
10	1011110101
01	0110101110
111000	000111

- Another name for binary is base-2 (pronounced "base two")

What is a decimal number?

- The numbers that we are used to seeing are called decimal numbers.
- decimal numbers consist of the digits from 0 (zero) through 9.
- The following are examples of decimal #'rs

3	76
15	32423234
890	53

- Another name for decimal numbers are base-10 (pronounced "base ten") numbers.

Equivalence of Binary and Decimal

- Every Binary number has a corresponding Decimal value (and vice versa)
- Examples:

Binary Number	Decimal Equivalent
1	1
10	2
11	3
…	…
1010111	87

The value of a binary number

- Even though they look exactly the same, the value of the **binary number, 101**, is ***different*** from the value of the **decimal number, 101**.
  - The value of the binary number, 101, is equal to the decimal number five (i.e. 5)
  - The value of the decimal number, 101, is equal to one hundred and one
- When you see a number that consists of only ones and zeroes, you must be told if it is a binary number or a decimal number.

Computers store information using binary numbers

All information on computers is stored as numbers

- **All information** that is processed by computers is converted in one way or another into a sequence of numbers. This includes
  - numeric information
  - textual information and
  - Pictures
- Therefore, if we can derive a way to store and retrieve numbers electronically this method can be used by computers to store and retrieve **any type of information.**

How a computer stores information

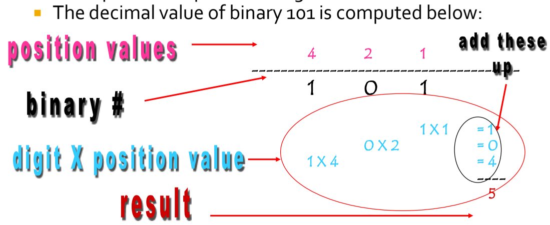
Binary Numbers are at the heart of how a computer stores all information

- Computers Store ALL information using Binary Numbers
- Computers use binary numbers in different ways to store different types of information.
- Common types of information that are stored by computers are :
  - Whole numbers (i.e. Integers).
    Examples: 8 97 -732 0 -5 etc
  - Numbers with decimal points.
    Examples: 3.5 -1.234 0.765 999.001 etc
  - Textual information (including letters, symbols and digits)
- Keep reading …

How to Convert from Binary to Decimal

Converting from binary to decimal

- Each position for a binary number has a value.
- For each digit, m`ultiply the digit by its position value
- Add up all of the products to get the final result
- The decimal value of binary 101 is computed below:



What about a longer number?

- In general, the "position values" in a binary number are the powers of two.
  - The first position value is 2^0 , i.e. one
  - The 2nd position value is 2^1 , i.e. two
  - The 2nd position value is 2^2 , i.e. four
  - The 2nd position value is 2^3 , i.e. eight
  - The 2nd position value is 2^4 , i.e. sixteen
  - etc.

Example

- The value of binary 01100001 is decimal 105. This is worked out below:

128	64	32	16	8	4	2	1	
0	1	1	0	1	0	0	1	

1 X 1 = 1
0 X 2 = 0
0 X 4 = 0
1 X 8 = 8
0 X 16 = 0
1 X 32 = 32
1 X 64 = 64
0 X 128 = 0

Answer: 105

Another example

- The value of binary 10011100 is decimal 156. This is worked out below:

128	64	32	16	8	4	2	1	
1	0	0	1	1	1	0	0	

0 X 1 = 0
0 X 2 = 0
1 X 4 = 4
1 X 8 = 8
1 X 16 = 16
0 X 32 = 0
0 X 64 = 0
1 X 128 = 128

Answer: 156

Some Terminology

- The following are some terms that are used in the computer field
  - Each digit of a binary number is called a **bit**.
  - A binary number with eight bits (i.e. digits) is called a **byte**.

How many different numbers?

- There are two different binary numbers with one bit:
  - 0
  - 1
- There are four different binary numbers with two bits:
  - 00 (i.e. decimal 0)
  - 01 (i.e. decimal 1)
  - 10 (i.e. decimal 2)
  - 11 (i.e. decimal 3)

How many different numbers?

- There are 8 different binary numbers with 3 bits:
  - 000 (i.e. decimal 0)
  - 001 (i.e. decimal 1)
  - 010 (i.e. decimal 2)
  - 011 (i.e. decimal 3)
  - 100 (i.e. decimal 4)
  - 101 (i.e. decimal 5)
  - 110 (i.e. decimal 6)
  - 111 (i.e. decimal 7)

different numbers - General Rule

# of bits	# of different binary numbers
1 bit:	$2^1 = 2$
2 bits:	$2^2 = 4$
3 bits:	$2^3 = 8$
4 bits:	$2^4 = 16$
5 bits:	$2^5 = 32$
6 bits:	$2^6 = 64$
7 bits:	$2^7 = 128$
8 bits:	$2^8 = 256$
9 bits:	$2^9 = 512$
10 bits:	$2^{10} = 1024$
etc.	

Smallest value for a binary

- The smallest value for a binary number of any number of bits is zero.
- This is the case when all bits are zero.

Smallest value for a binary

- The smallest value for a binary number with any number of bits is zero (i.e. when all the bits are zeros)

# of bits	smallest binary #	decimal value
1 bit:	0	0
2 bits:	00	0
3 bits:	000	0
4 bits:	0000	0
5 bits:	00000	0
6 bits:	000000	0
7 bits:	0000000	0
8 bits:	00000000	0
etc.		

Largest value for a binary

- The largest value for a binary number with a specific number of bits (i.e. digits) is when all of the bits are one.
- General rule: for a binary number with n bits, the largest possible value is : $2^n - 1$

Largest numbers

- The following are the largest values for binary numbers with a specific number of bits:

# of bits	largest binary #	decimal value
1 bit:	1	1
2 bits:	11	3
3 bits:	111	7
4 bits:	1111	15
5 bits:	11111	31
6 bits:	111111	63
7 bits:	1111111	127
8 bits:	11111111	255
etc.		

Why is it called "binary" (or base-2)?

- The prefix "bi" means "**two**" in Latin
- Binary derives its name from the fact that the digits in a "**Bi**nary" number can only have **two** possible values, 0 or 1
- It is also called "base-**2**" based on the fact that the column values are the powers of **2.** (i.e. 2^0 2^1 2^2 2^3 2^4 2^5 etc.)