Objectives

- Visualize the fundamental 3D geometric operations -- translation, rotation about the three coordinate axes, and scale about the origin
- Experimentally determine if 3D rotations commute
- Discover how to rotate about an arbitrary axis using a combination of coordinate axes rotations

Background

- Fundamental 3D geometric transformations: translation, rotation about the three coordinate axes, scale about the origin, as well as their associated 4x4 transformation matrices.

Software Tool

- Self-Training Tool for Learning 3D Geometrical Transformations (3gtd)
  
  http://www3.uji.es/~ribelles/

  Download it on your computer and proceed to the activities listed below.
Activities

Part 1: Getting Comfortable with the Software Tool

- Start the 3dgt software tool. The 3D scene is displayed in the middle. The menu on the right allows you to create and remove 3D primitives. The menu on the left allows you to apply different transformations to the selected object (which will be displayed in light yellow; to select an object, use Ctrl+Shift+Left mouse button.)

- To master the controls of this tool, begin by creating a torus and applying various transforms to it. For best viewing, try to adjust the scene so that it looks similar to that shown below.

![Scene Images]

Use the mouse and the keyboard to interact with the scene as follows:

- **Mouse Left** button for rotating the scene
- **Ctrl + Mouse Left** button for zooming
- **Shift + Mouse Left** button for panning
- **Ctrl + Shift + Mouse Left** button for selecting a primitive
- **Mouse Right** button for a popup menu with various options
- **Activity 1:** Start with a unit cube centered at the origin. Use two more cubes like this one and obtain the model shown in the right image, where each new cube is half the size of the previous one.

List the transformations below in the order in which they were applied:

- **Activity 2:** Start with a cone of base radius 1 and height 1, as shown in the left image. The right image shows the transformed cone of height 3 and a sphere of radius 1.

List the transformations below in the order in which they were applied:
Activity 3: Let's take a closer look at 3D rotations. Create a cone of radius 1 and height 3 as in the image below.

Devise an example involving two rotations such that the results are obviously different depending on the order in which the rotations are applied. List the two rotations below:

Activity 4: The left image shows a cone of base radius 1 and height 1. List the transformations necessary to convert the left cone into the right cone in the image.
Part 3: Rotations about arbitrary axes

There are situations in which rotations about axes other than the three coordinate axes are useful. In the next two activities, you will discover how a rotation about an arbitrary axis can be broken down into a series of rotations about the three coordinate axes.

Activity 5. The yellow axis passing through the teapot in the image below is parallel to the vector \((1, 1, 0)\). Determine a sequence of coordinate axis rotations that result in the teapot rotating 180 degrees about the yellow axis, as shown in the right image.

Write the sequence of rotations in the order in which you applied them:
Activity 6: In this activity, the yellow axis passing through the teapot is parallel to the vector \((4, 5, -3)\). Determine a sequence of coordinate axis rotations that results in the teapot rotating 90 degrees about the yellow axis, as shown below.

Write the sequence of rotations in the order in which you applied them.
**Activity 7:** Now let's consider rotating about an axis that does not pass through the origin. Here the yellow axis is parallel to the vector \((1, 0, 0)\) and passes through the point \((0,0,3)\). Determine a sequence of transformations that results in the teapot rotating 180 degrees about the yellow axis, as shown below.

Write the sequence of rotations in the order in which you applied them.