Computer Graphics – Hands-on

Two-Dimensional Transformations

Objectives

- Visualize the fundamental 2D geometric operations translation, rotation about the origin, and scale about the origin
- Experimentally and mathematically discover transformations that do and do not commute
- Learn how to compose transformations
- Discover how fixed point rotations and scales can be achieved by combining the fundamental operations

Background

- Fundamental 2D geometric transformations: translation, rotation about the origin, scale about the origin, as well as their associated 3x3 transformation matrices.

Software Tool

- Applet posted online

  http://www.csc.villanova.edu/~mdamian/graphics/applets/Transformations2D.html

  Courtesy of Dr. Robin Flatland, Siena College.

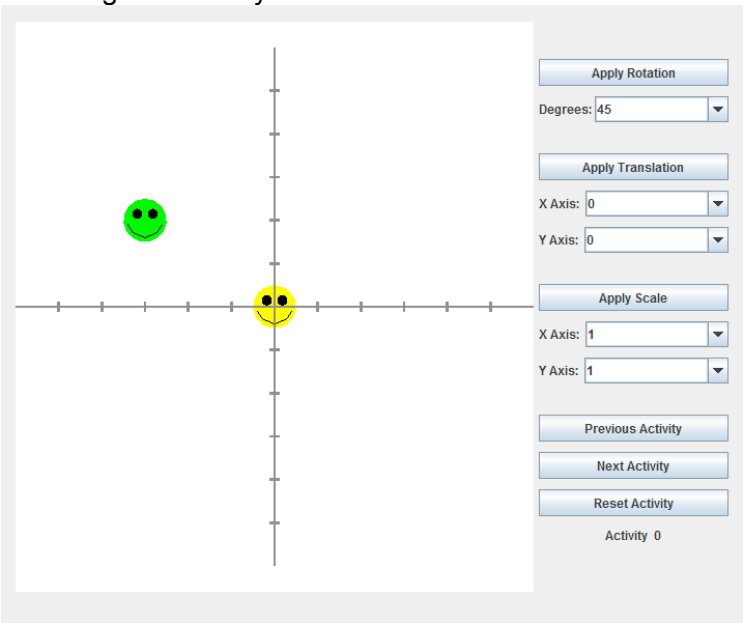
Activities

- Start the Transformations2D software tool. The buttons in the top right part of the window allow you to apply different transformations to the yellow smiley face. The buttons in the lower right allow you to step through the activities. The number of the current activity is displayed at the bottom. When you complete an activity, you will need to press the “Next Activity” button to advance. Some activities will display a green smiley face in addition to the yellow one. The goal of these activities is to transform the yellow face so that it has the same shape and position as the green one.

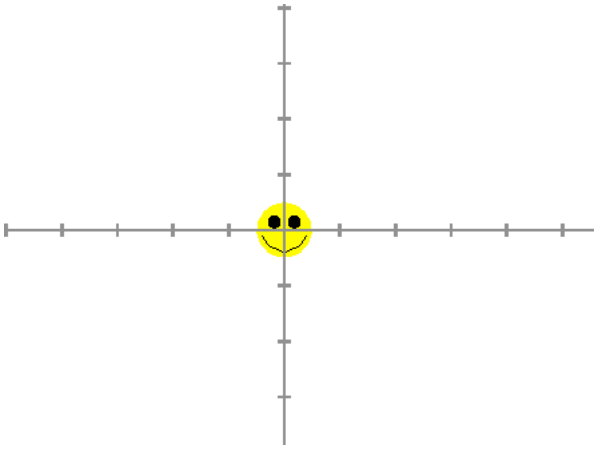
- We’ll begin with three easy warm-up activities to get comfortable with the controls and then move on to some more challenging activities. As you develop solutions to the posed problems, be sure to test your solutions using the tool to see if the results are what you are expecting.

Part 1: Warm-up Activities

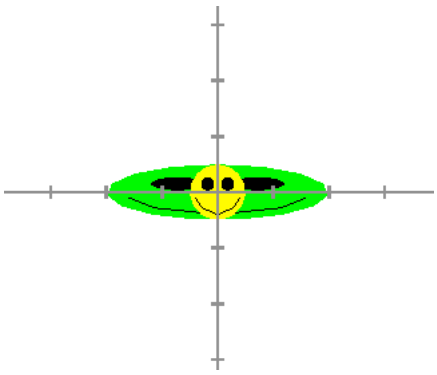
Activity 0: What transformation could you apply to the yellow smiley face to make it look like the green smiley face?



Activity 1: Click on the "Next Activity" button to advance to Activity 1. What transformation could you apply to the yellow smiley face so that it is upside down?



Activity 2: Click on the “Next Activity” button to advance to Activity 2. What transformation could you apply to the yellow smiley face to make it look like the green smiley face?



Below, give the 3x3 scale matrix (in homogeneous coordinates) that was applied to the points to make the yellow smiley face look like the green face.

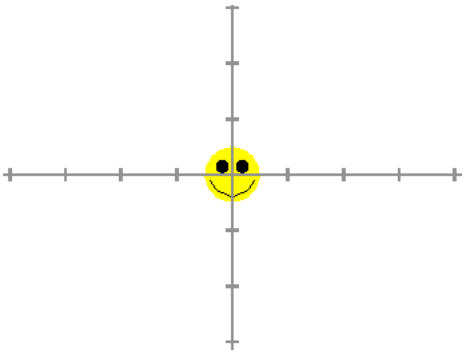
Reset activity 2 using the “Reset Activity” button. Before the scale operation is applied, notice that the top right tip of the yellow face’s smile is located at the point (0.4, -0.1). Now apply the scale operation again, making it look like the green face. Judging from the coordinate axes on the software tool, approximately where does this point end up after the scale operation is applied?

Below, mathematically compute the image of point (0.4, -0.1) under the scale operation by multiplying it (in homogeneous coordinates) by the 3x3 scale matrix. If the results are not consistent with what you see on the display, double check your scale matrix and your calculations.

Part 2: Discovering transformations that do and do not commute

Activity 3: Click the "Next Activity" button to advance to Activity 3. Now let's try some more challenging things. Sometimes when you apply two consecutive transformations, the order in which you apply them doesn't matter. For example, try scaling the yellow smiley face by 2 along both coordinate axes and then rotating it 45 degrees. Now reset the activity and apply the two transformations in the opposite order -- rotate first and then scale. You see that you get the same results either way. When the order doesn't matter, the operations are said to commute.

But for some transformation combinations, the order *does* make a difference in the result. Using the yellow smiley face, find two transformations such that the result is obviously different depending on the order in which they are applied. Below, write the two transformations you discovered that *do not* commute.



Below is a chart of the fundamental transformations. For each pair of transformations, try to find an example in which they don't commute using the software tool. For each pair in which you find such an example, write the letters NC in the box to indicate that they do not generally commute. For the entries down the diagonal, you will be looking for an example in which two of the same type of transformation do not commute.

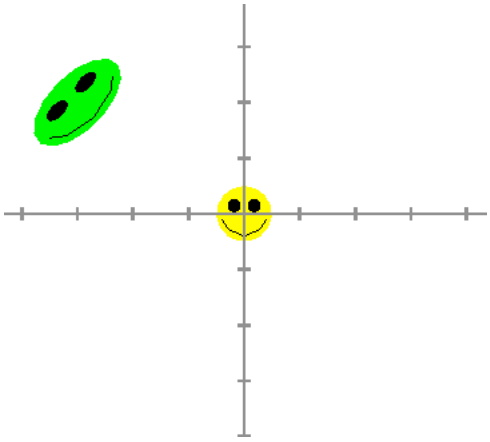
Rotation	Translation	Uniform Scale	Non-uniform Scale	
				Rotation
				Translation
				Uniform Scale
				Non-uniform Scale

Hint – there are four in all that don't commute.

Pick one of the boxes above in which you did not write NC. Just because you couldn't find an example in which the two transformations did not commute does not guarantee of course that no such example exists. Below, **prove** that the two transformations always commute by showing that the order in which you multiply the two transformation matrices doesn't matter. In other words, show that you get the same composite matrix either way.

Part 3: Composing Transformations

Activity 4: Determine a transformation or a sequence of transformations that, when applied to the yellow smiley face, transform it to look like the green smiley face. Below, list the transformations in the order in which you applied them.

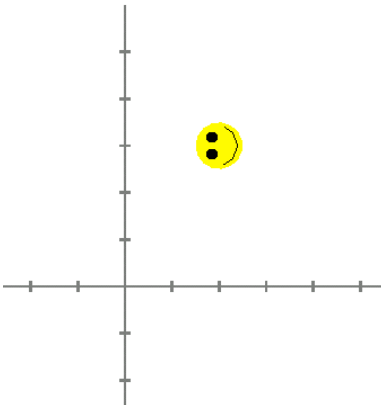


Below, calculate the 3x3 composite matrix that was applied to the points making up the yellow face, transforming it to look like the green face. Show your work.

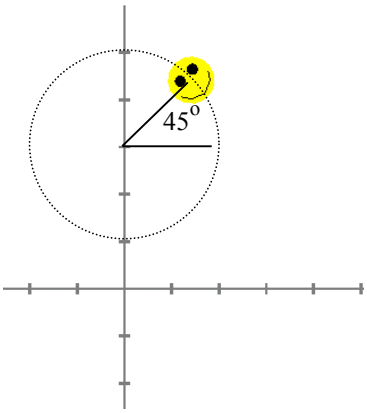
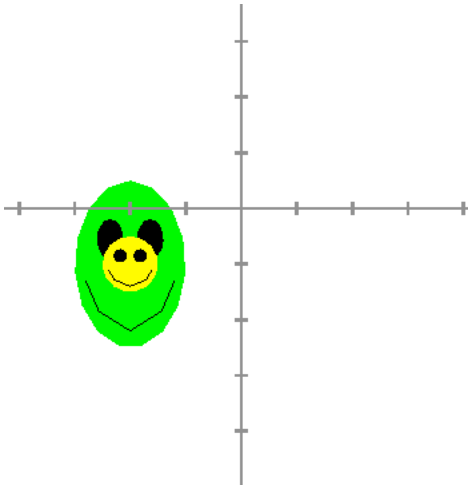
Reset the activity. Notice that, before the transformations are applied, the right side of the yellow face has coordinates (0.5, 0). Now apply the transformations again, making it look like the green face. Judging from the coordinate axes on the software tool, approximately where does the right side of the yellow face end up after the transformations are applied?

Below, mathematically compute the image of point (0.5, 0) under the transformations by multiplying it (in homogeneous coordinates) by the 3x3 composite matrix you computed. If the results are not consistent with what you see on the display, double check your calculations.

Activity 5: So far, all of the rotations we have considered have been about the origin. Here we consider rotating about some other point in the plane. In this activity, the yellow smiley face is centered at (2, 3). Determine a sequence of transformations that, when applied to the yellow smiley face, rotates it 90 degrees counter-clockwise about its center as shown below. In other words, the face rotates "in place." The point that an object is rotated about is call the *fixed point* or the *pivot point*. Here the fixed point is (2, 3). Below, list the transformations in the order in which you applied them.



Activity 6: In this activity the yellow smiley face is again centered at (2, 3). But here we want a sequence of transformations that will result in the yellow smiley face rotating 45 degrees counter-clockwise about the fixed point (0, 3), as shown below. Here the fixed point is neither the origin nor the center of the object! Below, list the transformations in the order in which you applied them.



Activity 7: So far we have only considered scaling objects about the origin. Here we consider how we can scale an object about some other fixed point in the plane. Determine a sequence of transformations that, when applied to the yellow smiley face, transforms it to look like the green smiley face. Note that both the yellow and green smiley faces are centered at (-2, -1), but the green smiley face has been stretched by a factor of 2 along the X axis and by a factor of 3 along the Y axis. Below, list the transformations in the order in which you applied them.