CSC 1300 – Problem Set 6

1. How many 6-bit strings have 1010 as the first four bits, have 101 as bits 2, 3, and 4, or have 10 for bits 3 and 4?

2. Assume that in a group of 6 people, each pair is either friends or enemies. Show that in this group there are at least three mutual friends or at least three mutual enemies.

3. How many area codes are needed to guarantee that 20 million phones in a state have distinct phone numbers? Note that phone numbers must be of the form \( EDD-EDD-DDDD \), where \( D \) can be any digit, but \( E = 2, \ldots, 9 \) (i.e., it cannot be a 0 or a 1); the first three digits are the area code.

4. Consider the alphabet A, B, C, D, E, F.
   (a) How many four-letter words contain the subword ACE?
   (b) How many four-letter words don’t begin with F or don’t end in E?
   (c) How many five-letter words contain the subword CAB?
   (d) How many four-letter words begin with C or end in two vowels?

5. Show that \( \binom{2n}{2} = 2 \binom{n}{2} + n^2 \).