CSC 1300 – Problem Set 3

1. (15 points) Consider the open sentence $E(x)$ denote that $x$ is an even number, with domain the natural numbers. Let $P$ denote the sentence:
   
   “For every even number $x$, the number $x^2$ is even. “

   a) Express $P$ as a quantified statement logical notation
   
   b) State the negation of $P$
      
      • using logical notation
      • in natural sounding English
   
   c) State the converse of $P$
      
      • using logical notation
      • in natural sounding English
   
   d) State the contrapositive of $P$
      
      • using logical notation
      • in natural sounding English

2. (20 points) Prove or disprove each of the statements in question 1.

3. (20 points) Consider the following possible formulas for summing up some positive odd numbers? Which one(s) are correct? Give a brief justification to support each answer (no formal proof necessary).

   a)  
   
   \[ \sum_{i=1}^{n} (2i - 1) = n^2 \]

   b)  
   
   \[ \sum_{i=0}^{n} (2i - 1) = n^2 \]

   c)  
   
   \[ \sum_{i=0}^{n} (2i + 1) = n^2 \]

   d)  
   
   \[ \sum_{i=1}^{n} (2i + 1) = n^2 \]

4. (15 points) Use mathematical induction to prove one of the above formulas.

5. (15 points) Use mathematical induction to prove:

   \[ 1 \cdot 2 + 2 \cdot 3 + 3 \cdot 4 + 4 \cdot 5 + \cdots + n(n + 1) = \frac{n(n+1)(n+2)}{3} \]

6. (15 points) Use mathematical induction to prove that $5^n - 1$ is divisible by 4 for every $n \in \mathbb{N}$. 