CSC 1300 – Problem Set 2

1. (8 points) Let \( X = \{a\} \). For each of the following sets, indicate whether \( X \) is an element of the set:
   (a) \( A = \{a, \{b\}\} \)  (b) \( B = \{b, \{a, b\}\} \)  (c) \( C = \{\{a, b\}, \{a\}\} \)  (d) \( D = \{\{a\}\} \)

2. (8 points) Let \( X = \{a\} \). For each of the following sets, indicate whether \( X \) is a subset of the set:
   (a) \( A = \{a, \{b\}\} \)  (b) \( B = \{b, \{a, b\}\} \)  (c) \( C = \{\{a, b\}, \{a\}\} \)  (d) \( D = \{\{a\}\} \)

3. (12 points) Let \( A = \{x \mid x \in \mathbb{Z} \text{ and } -3 \leq x \leq 1\} \) and \( C = \{x \in \mathbb{Z} \mid |x| < 4\} \). Find all sets \( B \) such that \( A \subset B \subset C \).

4. (12 points) How many elements are in \( \mathcal{P}(A) \) if \( A = \{x \in \mathbb{Z} \mid |x| < 5\} \)?

5. (12 points) Let \( S \) be the set of students at Villanova who have been your classmate in one or more courses and let \( T \) be the set of students with whom you have studied. Describe each of the following sets in terms of \( S \) and \( T \):
   (a) the set of students who have neither been your classmate nor with whom you have studied,
   (b) the set of students with whom you have studied but who have never been your classmate,
   (c) the set of classmates with whom you have never studied,
   (d) the set of students with whom you have either studied or have been your classmate but not both,

6. (12 points) Let \( S = \{a, b, c, d\} \).
   (a) Partition \( S \) into as many subsets as possible such that no two subsets have the same number of elements.
   (b) Partition \( \mathcal{P}(S) \) into as many subsets as possible such that no two subsets have the same number of elements.

7. (12 points) For the sets \( \mathbb{Z}, \mathbb{Q}, \mathbb{R}, \) and \( \mathbb{I} \) (the integers, rational numbers, real numbers, and irrational numbers, respectively), where \( \mathbb{R} \) is considered the universal set, determine the following:
   (a) \( \mathbb{Z} \cap \mathbb{Q} \)  (b) \( \mathbb{I} \cap \mathbb{Q} \)  (c) \( \mathbb{Z} \cup \mathbb{Q} \)  (d) \( \mathbb{Z} - \mathbb{Q} \)  (e) \( \overline{\mathbb{Q}} \)  (f) \( \mathbb{I} \cup \mathbb{Q} \)

8. (12 points) Let \( A = \{1, 2, 3\} \) and \( B = \{2, 4\} \). Let \( A 	imes B \) denote the Cartesian product of \( A \) and \( B \).
   (a) \( (A \cup B) \times (A \cap B) \)  (b) \( (A \times B) \cap (B \times A) \)  (c) \( (B - A) \times A \)  (d) \( (A \times A) \cap (B \times B) \)

9. (12 points) For sets \( A \) and \( B \), which of the following sets are equal? Explain your answer using Venn diagrams or set identities.
   (a) \( A \oplus B \)  (b) \( \emptyset \)  (c) \( (A \cap B) - (B \cup A) \)  (d) \( (A \cup B) - (B \cap A) \)  (e) \( (A - B) \cup (B - A) \)  (f) \( (A \oplus B) - (B \oplus A) \)