1. (20 points) Consider the following open sentences:

\[ P(x, y): x + y > 0. \quad Q(x, y): xy > 0. \]

(a) State \( P \lor Q \)
(b) State \( \neg(P) \lor Q \)
(c) State \( \neg(P \lor Q) \) using the phrase “it is not the case that”
(d) Use an appropriate De Morgan’s Law to restate \( \neg(P \lor Q) \)

2. (10 points) For integers \( x \) and \( y \), consider the biconditional:

\( x + y \) is even if and only if \( 3x \) and \( 5y \) are even

(a) Give an example of two distinct integers \( x \) and \( y \) for which this biconditional is true.
(b) Give an example of two distinct integers \( x \) and \( y \) for which this biconditional is false.

NOTE: This question is corrected here (originally, it was wrongly stated and made no sense).

3. (15 points) Let \( P, Q, R \) be statements. Prove the following are true using truth tables.

(a) \( P \lor (Q \land \neg R) \equiv (P \lor Q) \land (P \lor \neg R) \)
(b) \( P \rightarrow (Q \lor R) \equiv (P \rightarrow Q) \lor (P \rightarrow R) \)
(c) \( P \land (Q \oplus R) \equiv (P \land Q) \oplus (P \land R) \)

4. (35 points) Consider the implication: If \( x \) is prime and greater than 2, then \( x \) is odd.

(a) State the implication using the phrase “only if”
(b) State the implication using the word “sufficient”
(c) State the converse of the implication
(d) State the contrapositive of the implication
(e) Is the implication true for some positive integer \( x \)? Is it true for all positive integers \( x \)?
(f) Is the converse of the implication true for some positive integer \( x \)? Is it true for all positive integers \( x \)?
(d) Is the contrapositive of the implication true for some positive integer \( x \)? Is it true for all positive integers \( x \)?

5. (20 points) Let \( P \) and \( Q \) be statements. Determine whether the compound statements below are tautologies, contradictions, or neither. Justify your answer using truth tables or using one or more of the laws governing logical equivalences.

(a) \( (P \land \neg Q) \land (P \land Q) \)
(b) \( (P \land Q) \rightarrow (P \rightarrow Q) \)
(c) \( (P \land \neg Q) \rightarrow (P \lor Q) \)
(d) \( (d) (P \land Q) \leftrightarrow ((\neg P) \lor (\neg Q)) \)