Mathematical Induction

CSC 1300 – Discrete Structures
Villanova University
Example

For all natural numbers \( n \),

\[
1 + 2 + \cdots + n = \frac{n(n + 1)}{2}
\]
Mathematical Induction

• Use to prove universal statements of the form:
  – $\forall n \ P(n)$

• Easy to think of it as the domino effect
  Let $P(n)$ be the statement: domino $n$ is knocked over
  – If the first domino is knocked over ($P(1)$ is true); AND
  – if whenever the $k^{\text{th}}$ domino is knocked over,
    then the $(k + 1)^{\text{th}}$ is also knocked over
    i.e., $P(k) \rightarrow P(k + 1)$ is true for every $k$

..... It follows that all the dominoes are knocked over.
(i.e., $\forall n \ P(n)$ is true)

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Principle of Mathematical Induction

\[ [ P(1) \land \forall k (P(k) \rightarrow P(k + 1)) ] \rightarrow \forall n P(n) \]

- Base Case
- Inductive Hypothesis
- Inductive Consequent

Want to prove this
Examples

Generalized deMorgan’s Law

\[ n^3 - n \] is divisible by 3

Sum of powers of 2

\[ n! < n^n \]
Summation Notation

\[ \sum_{i=1}^{n} a_i \] denotes the sum \( a_1 + a_2 + \ldots + a_n \)

Example:

\[ \sum_{i=1}^{3} (5i + 10) = \]
Write the Summation Formula for ...

\[ 2 + 4 + 6 + 8 + 10 \]

\[ 2 + 2 + 2 + 2 + 2 \]

\[-1 + 1 + -1 + 1 + -1 + 1 \]

\[ 6 + 8 + 10 + 12 + 14 + 16 \]
Variations

\[ \sum_{i=1}^{n} a_i \]

\[ \sum_{j=1}^{4} f(j) \]

\[ \sum_{x \in \mathbb{N}} \frac{1}{x!} \]

\[ \sum_{i=0}^{2} a_i \]

\[ \sum_{k=1}^{3} (2k+1) \]

\[ \sum_{v \in V} \deg(v) = 2e \]
Compute this ...

\[ \sum_{i=2}^{4} i^2 = \]

\[ \sum_{k=5}^{5} k^3 = \]

\[ \sum_{i=1}^{2} \sum_{j=0}^{3} i j = \]
Some important summation formulas

\[ \sum_{i=1}^{n} 1 = 1 + 1 + \ldots + 1 = n \]

More generally, \( \sum_{i=a}^{b} 1 = b - a + 1 \)

\[ \sum_{i=1}^{n} i = 1 + 2 + \ldots + n = n(n+1)/2 \]

\[ \sum_{i=1}^{n} i^2 = 1^2 + 2^2 + \ldots + n^2 = n(n+1)(2n+1)/6 \]

\[ \sum_{i=0}^{n} 2^i = 2^0 + 2^1 + \ldots + 2^n = 2^{n+1} - 1 \]

More generally, \( \sum_{i=0}^{n} ar^i = a(r^{n+1} - 1)/(r - 1) \) \( (r \neq 1) \)
Basic summation rules

\[ \sum c a_i = c \sum a_i \]

\[ \sum (a_i \pm b_i) = \sum a_i \pm \sum b_i \]

\[ \sum_{i=k}^{n} x_i = \sum_{i=k}^{m} x_i + \sum_{i=m+1}^{n} x_i \]
Compute the sums

\[ \sum_{i=3}^{n+1} 1 = \]

\[ \sum_{i=2}^{n-2} i = \]

\[ \sum_{i=1}^{n-1} 2^i = \]

1 + 2 + 4 + … + 100 =

2 + 4 + 8 + … + 1024 =

9 + 16 + 25 + 36 + 49 + 64 + 81 + 100 =

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Try this

• Consider the sum: $1 + 3 + 5 + 7 + \cdots + (2n - 1)$

  – Write it in summation notation
  – Find a formula for the sum you wrote (consider using a picture?)
  – Prove your formula is correct using mathematical induction
Tiling with triominoes – try this!