CSC 1300 – Problem Set 1

1. (20 points) For an integer $x$ consider the two sentences:
   
   $P: x < -3$  $Q: x \geq -3$

   (a) State $P \land Q$
   (b) State $P \lor Q$
   (c) Use De Morgan's Laws to state the negation of (a).
   (d) Use De Morgan's Laws to state the negation of (b).

2. (25 points) Let $P, Q, R, S, U,$ and $W$ be statements. Prove the following are true. Justify your answer using truth tables or using one or more of the laws governing logical equivalences.

   (a) $P \lor (Q \lor \neg R) \equiv \neg R \lor (P \lor Q)$
   (b) $P \lor (Q \land \neg R) \equiv (P \lor Q) \land (P \lor \neg R)$
   (c) $P \rightarrow (Q \lor R) \equiv (P \rightarrow Q) \lor (P \rightarrow R)$
   (d) $P \land (Q \oplus R) \equiv (P \land Q) \oplus (P \land R)$
   (e) $P \land (\neg P \lor Q) \land (\neg Q \lor R) \land (\neg R \lor S) \land (\neg S \lor \neg U) \land (U \lor W) \equiv P \land Q \land R \land S \land \neg U \land W$

3. (35 points) Consider the implication: If $x$ and $y$ are even, then $xy$ is even.

   (a) State the implication using the phrase “only if”
   (b) State the implication using the word “sufficient”
   (c) State the converse of the implication
   (d) State the contrapositive of the implication
   (e) Is the implication true for some positive integers $x$ and $y$? Is it true for all positive integers $x$ and $y$?
   (f) Is the converse of the implication true for some positive integers $x$ and $y$? Is it true for all positive integers $x$ and $y$?
   (e) Is the contrapositive of the implication true for some positive integers $x$ and $y$? Is it true for all positive integers $x$ and $y$?

4. (20 points) Let $P$ and $Q$ be statements. Determine whether the compound statements below are tautologies, contradictions, or neither. Justify your answer using truth tables or using one or more of the laws governing logical equivalences.

   (a) $(P \land \neg (Q)) \land (P \land Q)$
   (b) $(P \land Q) \rightarrow (P \rightarrow Q)$
   (c) $(P \land \neg (Q)) \rightarrow (P \lor Q)$
   (d) $(P \land Q) \leftrightarrow ((\neg P) \lor (\neg Q))$