CSC 1300 – Problem Set 5

1. The members of the Villanova University Dance Team will be participating in the National Dance Alliance competition in Daytona Beach, Florida. Expectations are running high, since this awesome team has placed top 10 in Division I Open Dance and Division I Hip Hop over the past years in the national competition! To prepare for the competition, one of the team members is pondering a particular piece that they are performing, involving six members of the team: Amanda, Brianna, Christina, Daphne, Ellie, Frances.

a) How many ways can you line up the dancers?
b) How many ways can you line up the dancers if Daphne or Ellie must be on the leftmost position?
c) How many ways can you line up the dancers if Brianna or Amanda must be on the rightmost position?
d) How many ways can you line up the dancers if Daphne or Ellie must be on the leftmost position and Brianna or Amanda must be on the rightmost position?
e) How many ways can you line up the dancers if neither Daphne nor Ellie can be in the leftmost position and neither Brianna nor Amanda can be in the rightmost position?

2. The dance team is setting up a three-member committee to address difficult combinatorial questions. They have calculated that there are 455 ways to form the committee. However, the concern is expressed that not all the team members have taken CSC 1300, so they might not be prepared to serve on this committee. In fact, of the 455 possible committees, 220 have exactly one member with adequate mathematical background, 66 have exactly two, and only 4 have all three members up to the job!

a) How many committees have a majority of members with adequate preparation?
b) How many committees have nobody with adequate preparation?
c) How many committees have at most two members with adequate preparation?
d) Challenge: How many members does the Villanova Dance Team have? And how many of those do you think have taken CSC 1300 or some other course that prepares them to address all the combinatorial challenges that lie ahead?

3. Consider the equation:

\[ x_1 + x_2 + x_3 + x_4 + x_5 + x_6 + x_7 = 15 \]

a) How many integer solutions are there, if \( x_i \geq 0 \), for \( 1 \leq i \leq 7 \)?
b) How many integer solutions are there, if \( x_i \geq 1 \), for \( 1 \leq i \leq 7 \)?
c) How many integer solutions are there, if \( x_i \geq 2 \), for \( 1 \leq i \leq 7 \)?
4. Let’s count banana splits. These are ice-cream treats that have three scoops of ice cream (two or three of the scoops could be the same flavor), three toppings (two or three of the toppings could be the same flavor), whipped cream (always), a choice of nuts or no nuts, and a choice of a cherry or no cherry, all placed atop two banana halves. If there are 18 different flavors of ice cream and 5 choices of toppings, how many different banana split orders are possible? Note that people do care which toppings end up on which scoops, so the positions of the scoops should be labeled.

5. Let us say that two words are equivalent if they are anagrams of each other.
   a) How many seven-letter words are equivalent to THOUGHT? (Note that the words do not have to be real words or make sense.)
   b) Is this an equivalence relation on the set of all words?

6. The Computing Sciences department is preparing a report for accreditation of the undergraduate CSC major. Dr. Cassel needs your help in double-checking the report. The report states that 119 students took Calculus I in the Fall 2015 semester. The report notes that, in Spring 2016, 96 of these students took Calculus II, 53 of them took Discrete Mathematics, and 39 of them took Physics II. The report also says that 38 of the students took both Calculus II and Discrete Mathematics, 31 of the students took both Discrete Mathematics and Physics II, 32 of the students took both Calculus II and Physics II, and 22 of the students took all three courses. You examine the report and find a discrepancy. Explain.