CSC 1300 – Problem Set 3

1. Consider the sum: \( 2 + 4 + 6 + \cdots + 100 \)

   a) Express it as a summation.

   b) Generalize this summation to express the sum of the first \( n \) positive even numbers and conjecture a formula for this sum.

   c) Prove your formula using mathematical induction.

2. Use mathematical induction to prove:
   \[
   1 \cdot 2 + 2 \cdot 3 + 3 \cdot 4 + 4 \cdot 5 + \cdots + n(n+1) = \frac{n(n+1)(n+2)}{3}
   \]

3. Consider the inequality \( 2^n > n^2 \). For which values of \( n \) is this true? Prove your answer using mathematical induction.

4. Use mathematical induction to prove that \( 5^n - 1 \) is divisible by 4 for every \( n \in \mathbb{N} \).

5. Use mathematical induction to prove the generalized distributive property for sets:
   \[
   (A_1 \cup A_2 \cup \cdots \cup A_n) \cap B = (A_1 \cap B) \cup (A_2 \cap B) \cup \cdots (A_n \cap B)
   \]
   for every \( n \in \mathbb{N} \), \( n \geq 2 \).

   Note: You can make of the distributive property for two sets without proving it:
   \[
   (X \cup Y) \cap Z = (X \cap Z) \cup (Y \cap Z)
   \]