CSC 1300 – Problem Set 2

1. Let $A = \{1, 2, 3\}$ and $B = \{3, 4, 5\}$. List the elements of...

   (a) $(A \cup B) \times (A \cap B)$
   (b) $(A \times B) \cap (B \times A)$
   (c) $(B \setminus A) \times A$
   (d) $(A \times A) \cap (B \times B)$

2. Prove that $A \subseteq B$ iff $A \cup B = B$.

3. Planning for the School of Rock tribute to the Rolling Stones is at full swing with practices from 4-6pm every day this week. The guitar players, however, need a lot more practice! Their teacher Paul decides to end the group’s Wednesday’s practice at 5pm so he can focus on the guitar players to make sure they are ready for the show. The rest of the group can go around to check the equipment and arrange some props in the auditorium to get ready for the show, and then go out for ice cream. At the last minute, the boys among the guitar players decide they want to practice alone (girls not invited!). Surprisingly, the girls agree (they don’t need the extra practice, so there!). Paul reluctantly agrees, so it is determined that the Wednesday practice is for guitar players, but with girls excluded.

   • Anna asks: “So, all the girls get to go for ice cream?”
   • Ben jumps in: “Not just girls, anyone who is not a guitar player”
   • Connor corrects them: “It is the girls who are not guitar players who get to go.”
   • Danika disagrees: “Anyone who is a girl or not a guitar player gets to go”

   a) Assuming the universal set is the set of all children taking part in the Rolling Stones tribute, represent the girls (G), boys (B), and guitar players (P) as sets and then use set notation and Venn diagrams to express the set of children doing the extra practice (X) and children going for ice cream (I) in terms of G, B, and P. Be sure to draw and label your Venn diagrams so as to demonstrate how these sets are related.

   b) Express the statements made by Anna, Ben, Connor, and Danika in terms of sets and explain which one(s) are right and why. Refer to the Venn Diagrams as necessary.

   c) In your answer for (b) you may have made use of the assumption that all the children are binary at this point (i.e., they identify as either boy or girl).

      i. Express this assumption using sets.
      ii. Examine your answer in (b) and identify where that assumption might be relevant.
4. Let:

\( g \): You will graduate from Villanova after this semester
\( m \): You have fulfilled major and core requirements
\( c \): You are a CS major
\( p \): You have completed at least 40 courses of 3 credits or more

(a) Express each of the sentences below using \( g, m, c, p \), and logical connectives.

A. In order to graduate from Villanova at the end of this semester, you need to satisfy your major and core requirements, and to have completed at least 40 courses of 3 credits or more.

B. If you are a CS major and have fulfilled the major and core requirements and have done so by completing 40 courses of 3 credits or more, then you will graduate from Villanova at the end of this semester.

C. You will graduate from Villanova at the end of this semester if and only if you fulfill all the requirements and have completed at least 40 courses of 3 credits each.

D. If you are a CS major and have not completed 40 courses of 3 credits or more, you will not graduate from Villanova after this semester.

E. If you have completed 40 courses of 3 credits or more, then you have fulfilled all the major and core requirements.

F. If you are a CS major and have fulfilled all the major and core requirements, then you will graduate from Villanova at the end of this semester.

G. If you are a CS major and have not completed 40 courses of 3 credits or more, then you can’t have fulfilled all the major and core requirements.

H. If you are a CS major, you need complete 40 courses of 3 credits or more in order to fulfill all the major and core requirements.

(b) Repeat the previous question using sets, and draw Venn diagrams that exhibit the relations among sets expressed in each of the above sentences.

(c) Assume the proposition \( c \) (“you are a CS major”), above, is true and create a truth table of the remaining propositions (thus, we have just three, instead of four propositions in the truth table, making it more manageable). Compute truth table column for each of the above statements. Are any of the statements equivalent? Are any the converse or contrapositive of another statement?

(d) Bonus: Talk with some CS seniors and find out how this question is relevant to them. Which of the above statements are true? Which used to be true but are no longer true?

5. Write this in English (or, if you prefer, in Greek): \( \forall k \in \mathbb{Z}, \exists S \subseteq \mathbb{N}, |S| = k^2 \). Is it true? What is the negation of this statement? Write the negation using logical notation and in English. Is the negation true? Justify your answer.