Introduction

CSC 1300 – Discrete Structures

Villanova University

Discrete Structures

Goal: Understand how to use mathematics to reason about problems in Computer Sciences:
• sets
• functions and relations
• sequences
• summations
• logic
• proofs, including mathematical induction
• recurrences
• elementary combinatorics
• matrices
• trees
• graphs

What is “discrete” about Discrete Structures?

The word “discrete” by definition means distinct or separate entity. Therefore, Discrete Mathematics deals mainly with the analysis of a finite collections of objects. In other words, it is used whenever objects are counted and when a process involving a finite number of steps are analyzed. Discrete Math is often referred to as Finite Mathematics.

Listed below are just some of the computing topics that are finite by nature and require the concepts of Discrete Mathematics:
• Algorithms
• Data structures
• Databases
• Operating Systems
• Computer Security
• Digital imaging

Unlike Calculus, Discrete Mathematics is not concerned with infinite processes and does not support the notion of continuity. While in computing we sometimes “witness” infinite processes, in reality the need is for finite processing.

Examples of problems solved using discrete structures

• How many ways are there to choose a valid password?
• How can it be proven that a sorting algorithm correctly sorts a list?
• Is there a link between two computer systems in a network?
• How many habitats do you need to create in a zoo so that animals don’t eat each other?
Course organization

Approximately one chapter/week
• Before Tuesday:
  – write up solutions to problems assigned the previous week
  – read intro section of next Chapter, do practice exercises
• Tuesday in class: Quiz, do more exercises, in groups
• Before Thursday: read rest of chapter, do practice exercises
• Thursday in class: work on rest of chapter

An old quote

A priest asked: What is Fate, Master?
And he answered:
It is that which gives a beast of burden its reason for existence.
It is that which men in former times had to bear upon their backs.
It is that which has caused nations to build byways from City to City upon which carts and coaches pass, and alongside which inns have come to be built to stave off Hunger, Thirst and Weariness.
And that is Fate? said the priest.
Fate...I thought you said Freight, responded the Master.
That's all right, said the priest. I wanted to know what Freight was too.

- Kehlog Albran

Source unknown: This quote appeared as one of the "fortunes" displayed by the fortune cookie program on old unix systems. "fortune" was a program that ran automatically every time you logged out of the system and displayed a random, pithy saying.

Major Themes

• Counting
  – Sum Principle
  – Product Principle
  – Pigeonhole Principle
• Proofs
  – Direct
  – Counterexample

Sets and cardinality

Let $A = \{a, b, c\}, \quad B = \{1, 2\}$

The **cardinality** of a set $= \text{number of members}$

$|A| = 3$
$|B| = 2$

$A \cup B = \{a, b, c, 1, 2\} \quad A \cap B = \emptyset$

$(A \text{ and } B \text{ are disjoint})$

**Sum Principle:**
$|A \cup B| = |A| + |B|$
**Cartesian product**

Let \( A = \{a, b, c\}, \ B = \{1, 2\} \)

The *cartesian product* is the set of ordered pairs \((x, y)\) where \(x \in A\) and \(y \in B\):

\[
A \times B = \{(a, 1), (a, 2), (b, 1), (b, 2), (c, 1), (c, 2)\}
\]

**Product Principle:** \(|A \times B| = |A| \cdot |B|\)

---

**Sum Principle**

- Cardinality of disjoint union of sets
- Choose an element from *one* of several sets

**Sum Principle:** \(|A \cup B \cup C| = |A| + |B| + |C|\)

---

**Product Principle**

- Cardinality of cartesian product of sets
- Choose an element from *each* of several sets

**Product Principle:** \(|A \times B \times C| = |A| \cdot |B| \cdot |C|\)

---

**Problem #6 - 1.7**

A local creperie offers sweet crepes and savory crepes. A sweet crepe could have any fruit (banana, strawberry, mango, apple, lemon) and any syrup (nutella, chocolate, caramel, honey). A savory crepe could have any vegetable (broccoli, mushroom, spinach) and any protein (turkey, cheese, prosciutto). How many different crepes are on the menu?
But, what is Cardinality, exactly?

- 1-1 correspondance with natural numbers:

\[ |C| = 3 \]

Pigeonhole principle

- \( n \) pigeonholes: 1 2 3 .... \( n \)
- \( p \) pigeons: 1 2 3 .... ? ..... \( p \)
- if \( p > n \), then at least one pigeonhole has more than one pigeon
- if \( p > kn \), then at least one pigeonhole has more than \( k \) pigeons

Pigeonhole principle

- How many students do you need to have in a class to ensure that you have two born on the same month? week? same birthday?
- How about if you want to have at least 10 born on the same month?
- Example 1.5.5, p18

Chapter 1 Major Themes

- Counting
  - Sum Principle \(|A \cup B \cup ... \cup N| = |A| + |B| + ... + |N|\)
  - Product Principle \(|A \times B \times ... \times N| = |A| \cdot |B| \cdot ... \cdot |N|\)
  - Pigeonhole Principle \( n \) pigeonholes ..... \( p \) pigeons
    - \( p > n \) pigeons \( \Rightarrow \) at least one pigeonhole has more than one pigeon
    - \( p > kn \) \( \Rightarrow \) at least one hole with more than \( k \) pigeons.
- Proofs
  - Direct start from premise, reason to conclusion
  - Counterexample find an instance where the statement is not true