PBL 7: Discovering Permutations; Modeling A Genome

Pevzner suggests (p. 178ff) that the order of genes in an organism is represented by a permutation, presumably of an originally defined order that can be represented as the \( n \)-tuple \((1, 2, 3, \ldots, n)\). The current order could be written \((\pi_1, \pi_2, \ldots, \pi_n)\), where \(\pi_i\) is in \(\{1, 2, \ldots, n\}\) and \(\pi_i \neq \pi_j\) if \(i \neq j\).

He defines a particular kind of permutation on \(n\) things, which he calls a *reversal*, and is also known as an *inversion* in the realm of biology, but not in the realm of mathematics. A reversal changes the order of the elements of an \(n\)-tuple by writing \((\pi_1, \pi_2, \ldots, \pi_i, \pi_{i+1}, \ldots, \pi_j, \pi_{j-1}, \pi_j, \ldots, \pi_n)\) as \((\pi_1, \pi_2, \ldots, \pi_j, \pi_{j-1}, \ldots, \pi_{i+1}, \pi_i, \ldots, \pi_n)\), where \(i < j\).

To study the effects of reversals and the model of a genome as a permutation, you should investigate the following questions.

1. How many permutations of \(n\) things are there? How many that leave one element of an \(n\)-tuple fixed?

2. The set of permutations on \(n\) things is called the symmetric group of order \(n\), and denoted \(S_n\). Show how to construct all the elements of \(S_n\) given the elements of \(S_{n-1}\). Use this scheme to write out the elements of \(S_2\), \(S_3\), \(S_4\), \(S_5\) as \(n\)-tuples, given that \(S_1 = \{(1)\}\).

3. What does it mean to say that \(S_n\) is a group?

4. What does it mean to say that \(S_n\) is generated by a set of permutations?

5. Which permutations of \(S_4\) are obtainable using a single reversal? Can all permutations of \(S_4\) be obtained using a sequence of reversals? How about \(S_5\)? \(S_n\)?

6. What biological mechanism supports the idea that reversals are an important concept to model?

7. Using the breakpoint graph technique, find the reversal distance and a corresponding sequence of reversals necessary to transform the sequence of genes in the third chromosome of the Olympic strain of *Drosophila pseudoobscura* into that of the Standard strain (see Dobzhansky and Sturtevant, 1938).