Name: ANSWER KEY

VILLANOVA UNIVERSITY
Department of Computing Sciences
CSC 1300 – 002 February 2, 2016

Exam 1

Show your work carefully. Just writing an answer will not do. Show any assumptions; show the steps you took, and show how you came to your answer. You may refer to the card you brought with your notes. No other resource is permitted – no other notes, no calculators, no electronic devices.

I. (10 points) Sonny's Simple Sandwich Station offers three kinds of rolls (Italian, whole wheat, gluten-free), two kinds of cheese (provolone or Swiss), four types of vegetables (cucumbers, olives, tomatoes, peppers), and five kinds of meat (salami, ham, turkey breast, bologna, roast beef). The vegetable and meat are optional, so each includes an option of "none of the above." Your order must specify one from each category (remembering that the meat and vegetable lists also include a "none" option). How many different sandwich orders are possible?

rolls → 3 choices
cheese → 2 choices
veggies → 4+1=5 choices
meat → 5+1=6 choices

independent choices so multiply:

$$3 \cdot 2 \cdot 5 \cdot 6 = 180$$

II. (5 points) How many possible 8 digit Villanova ID numbers end in 666 and do not start with 6?

$$\underset{\uparrow\ 9\ \text{choices}}{\underline{a}}\ \ \underbrace{\underline{b}\ \ \underline{c}\ \ \underline{d}\ \ \underline{e}}_{10\ \text{choices each}}\ \ \underbrace{\underline{6}\ \ \underline{6}\ \ \underline{6}}_{\text{fixed}}$$

$$9 \cdot 10 \cdot 10 \cdot 10 \cdot 10 \cdot 1 \cdot 1 \cdot 1 = 90000$$

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III. (10 points)
(a) Is the product of two odd numbers even?
(b) Is the product of two odd numbers odd?
Give a counter example for one, and a proof for the other.

(a) $3 \cdot 5 = 15$ 15 odd so counterexample to (a).

(b) Let $n_1 = 2k_1+1$ and $n_2 = 2k_2+1$ be odd.

Then $n_1 \cdot n_2 = (2k_1+1)(2k_2+1)$

$= 4k_1k_2 + 2k_1 + 2k_2 + 1$

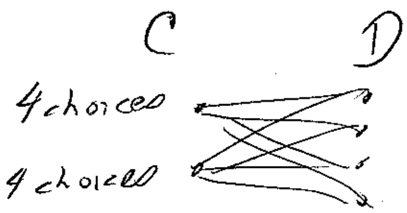
$= 2[2k_1k_2 + k_1 + k_2] + 1 = 2K+1$

which is odd. $\boxed{QED}$

IV. (5 points) Let C = {1, R2D2} and D = {M, BB8, {3,4}, 3}
a. List the elements of $C \times D$

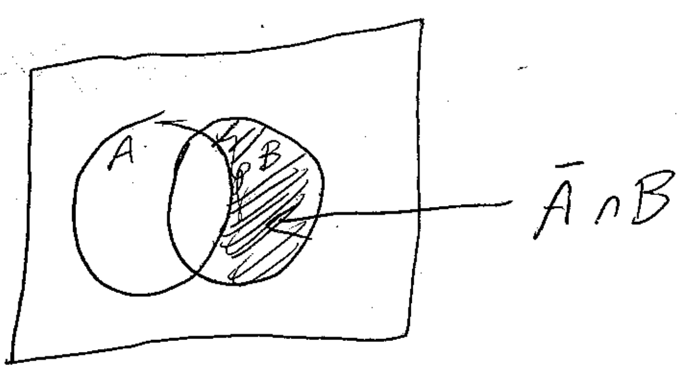
$\{(1, M), (1, BB8), (1, \{3,4\}), (1, 3), (R2D2, M), (R2D2, BB8), (R2D2, \{3,4\}), (R2D2, 3)\}$

b. How many functions are there to map elements of C to elements of D?



$4 \cdot 4 = 4^2 = 16$ different maps

V. (5 points) Make a Venn diagram that represents $\bar{A} \cap B$



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VI. (5 points) Use DeMorgan's laws to simplify $\overline{B \cup (\bar{C} \cap D)}$.

$$\overline{B \cup (\bar{C} \cap D)} = \bar{B} \cap \overline{(\bar{C} \cap D)}$$

$$\overline{\bar{C} \cap D} = \bar{\bar{C}} \cup \bar{D} = C \cup \bar{D}$$

so $\overline{B \cup (\bar{C} \cap D)} = \bar{B} \cap (C \cup \bar{D})$

VII. (10 points) Use a truth table to determine if the following two expressions are equal:
$B \Rightarrow (\neg C \vee D)$ and $(B \wedge C) \Rightarrow D$

B	C	D	$\neg C \vee D$	$B \Rightarrow (\neg C \vee D)$	$B \wedge C$	$(B \wedge C) \Rightarrow D$
T	T	T	T	T	T	T
T	T	F	F	F	T	F
T	F	T	T	T	F	T
T	F	F	T	T	F	T
F	T	T	T	T	F	T
F	T	F	F	T	F	T
F	F	T	T	T	F	T
F	F	F	T	T	F	T

the same so equal

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VIII. (10 points) $(n) = \lfloor \pi n \rfloor - 2$. Domain is N, target space N

i. Find *f(3)*.

$f(3) = \lfloor \pi 3 \rfloor - 2 = \lfloor 9.42478 \rfloor - 2 = 9 - 2 \equiv 7$

ii. Is *f(n)* one-to-one? (Explain)

Yes. Since $\pi = 3.14\ldots > 1$, each $\lfloor \pi n \rfloor > \lfloor \pi(n-1) \rfloor > \lfloor \pi(n-2) \rfloor \ldots$

so if $\lfloor \pi k_1 \rfloor - 2 = \lfloor \pi k_2 \rfloor - 2$ then $k_1 = k_2$.

iii. Is *f(n)* onto? (Explain) No

$f(1) = 1$ $f(2) = 4$ $f(3) = 7$ and every other $f(n) > 7$

so $f(n) = 2$ is impossible, so it is not onto.

IX. (5 points) Negate the following statement: $\exists a \in N \mid \forall b \in N, b = a + 2$.

$\neg\left[(\exists a \in \mathbb{N})(\forall b \in \mathbb{N})(b = a+2)\right]$

$(\forall a \in \mathbb{N})\, \neg\left[(\forall b \in \mathbb{N})(b = a+2)\right]$

$(\forall a \in \mathbb{N})(\exists b \in \mathbb{N})\, \neg(b = a+2)$

$(\forall a \in \mathbb{N})(\exists b \in \mathbb{N})(b \neq a+2)$

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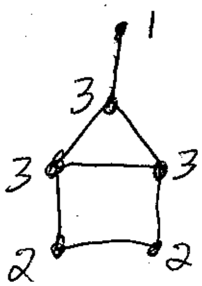
X. (5 points) Given the following truth table, write the Disjunctive Normal Form of the function.

P	Q	R	f(P,Q,R)
T	T	T	F
T	T	F	F
T	F	T	T
T	F	F	F
F	T	T	F
F	T	F	F
F	F	T	T
F	F	F	F

$(P \wedge \neg Q \wedge R) \vee (\neg P \wedge \neg Q \wedge R)$

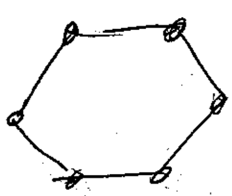
XI. (5 points) Draw a graph with degree sequence (1, 2, 2, 3, 3, 3).



many other possible answers

XII.

(a.) (5 points) Sketch C_6. Is it n-regular? If so, for what n? If not, why not?



Each vertex has degree 2 so it is n=2-regular.

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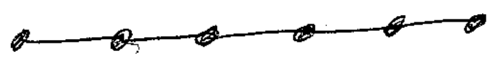
(b.) (5 points) Sketch P_6. Is it n-regular? If so, for what n? If not, why not?



Since this graph has two different vertices with unequal degrees, it is not n-regular

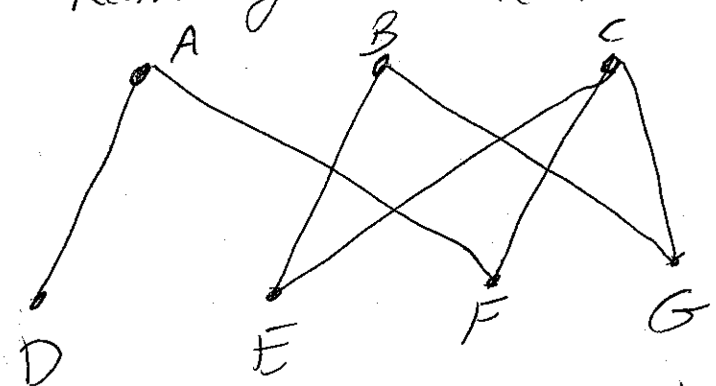
XIII. Consider the graph below:

(a.) (5 points) Show the adjacency matrix for this graph.

	A	B	C	D	E	F	G
A	0	0	0	1	0	1	0
B	0	0	0	0	1		1
C	0	0	0	0	1	1	1
D	1	0	0	0	0	0	0
E	0	1	1	0	0	0	0
F	1	0	1	0	0	0	0
G	0	1	1	0	0	0	0

(b.) (5 points) Show that this graph is bipartite

Rearrange the vertices to get



So $\{A, B, C\}$ has no edges between them and $\{D, E, F, G\}$ has no edges between them.

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(c.) (5 points) Verify the Handshaking Lemma for this graph.

vertex	degree
A	2
B	2
C	3
D	1
E	2
F	2
G	2

total 14

sum of degrees = 14 = 2(7)
where 7 = # of vertices.

Handshaking Lemma The sum of the degrees is twice the number of edges.