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VILLANOVA UNIVERSITY
Department of Computing Sciences
CSC 1300 – Final Exam
Spring 2016

Show your work carefully. Just writing an answer will not do. Show any and all assumptions; show the steps you took, and show clearly how you came to your answer.

If we have to look all over a page to try to find something that is part of the solution to the problem, you will be out of luck. Do scrap work on separate paper and do a neat and organized presentation of what you want to be graded**. Initial here** to show that you have read and understood this. _____.

The following items are provided in case you need them.

How many ways ... ... n labeled boxes?	at most one per box	any number per box	exactly one per box
k labeled (ordered) balls	A: $\binom{n}{k}k! = n(n-1)\ldots(n-k+1)$	E, F: (k_j balls unordered within box) $\frac{k!}{k_1!k_2!\ldots k_n!} = \binom{k}{k_1}\binom{k-k_1}{k_2}\binom{k-k_1-k_2}{k_3}\ldots\binom{k_n+k_{n-1}}{k_{n-1}}$	———
k unlabeled (unordered) balls	B: $\binom{n}{k}$	D, D': $\binom{k+n-1}{k} = \binom{k+n-1}{n-1}$ and $\binom{k-1}{n-1} = \binom{k-1}{k-n}$	———
unlimited balls, k different labels (order matters)	———	———	C: k^n

a	b	c	d	e	f	g	h	i	j	k	l	m	n	o	p	q	r	s	t	u	v	w	x	y	z
0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25

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1. Let A = {b, {c}} B = {D, EE, {$}}
   a. [_____/9] Find the following sets:
      i. A x B

      ii. $\mathcal{P}(B) =$ (that's the "power set" of B)

   b. [_____/3] How many functions *f* are there, *f:* A → B

   c. [_____/ 10] Enumerate all those functions (number them f_1, f_2, ..) by giving their action on each element of the domain. (Show the mapping from each element of the domain to something in the target space.)

   d. [_____/ 5] Which of those functions are 1:1? If none, explain why.

   e. [_____/ 5] Which of those functions are onto? If none, explain why.

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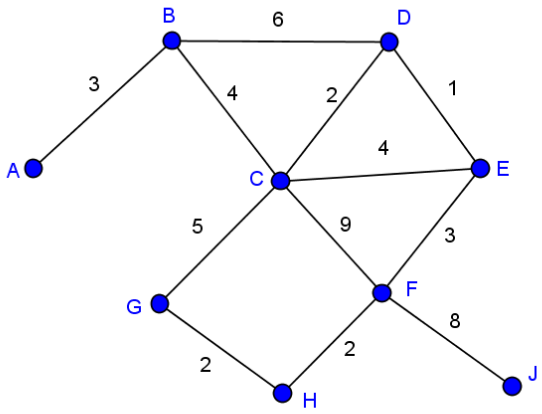
2. [_____/8] Given this truth table, write the Disjunctive Normal Form (DNF) for the formula *f* (P, Q, R)

P	Q	R	f(P,Q,R)
T	T	T	T
T	T	F	F
T	F	T	T
T	F	F	T
F	T	T	F
F	T	F	T
F	F	T	F
F	F	F	F

3. [_____/10] Show the adjacency matrix for this graph:



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4. [_____/12] For each definition of x~y, state whether it's an equivalence relation or not. If not, illustrate each property that fails.
    a. On the set of all Villanova students, x~y iff there was a class that x and y were in together.

    b. On **N**, x~y iff x (exactly) divides y.

    c. On **N**, x~y iff x+y is even.

5. About modular arithmetic
    a. [____/4] True or false? Explain your answer
        i. $-8 \equiv 31 \ (mod\ 13)$

        ii. $31 \equiv 31 \ (mod\ 13)$

    b. [_____/10] Given $a \equiv b \ (mod\ k)$ and $c \equiv d \ (mod\ k)$, prove $ac \equiv bd \ (mod\ k)$

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6. Induction and summations
   a. [____/5] Write the summation $\sum_{j=1}^{5}(2j-1)$ in expanded form, and calculate its value.

   b. [____/10] Prove by induction that for every natural number n,
   $$\sum_{j=1}^{n}(2j-1) = n^2$$

Be very careful to include all expected labels and statements relevant to the proof.

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7. Pascal's triangle and the Binomial Theorem
    a. [____ /5] Draw the first 7 rows of Pascal's triangle

    b. [_____ / 5] What do the values in row 6 represent with respect to choice notation?

    c. [_____ / 10] What is the term that contains y^4 in the expansion of

$$(3x - 6y)^7$$

    Show both the formula and the simplified value (ie, no binomial coefficient, no denominators).

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8. Next week we hope the softball team will win the Big East championship (12-5 this season in conference play). There are 17 CSC majors who will volunteer to help with the victory parade in downtown Philadelphia after the softball team wins the Big East.
   a. [/5] How many ways are there to select four students from this group of 17 majors to: 1) contact the media for coverage; 2) to set up the parade information homepage; 3) to write an article for the Inquirer; and 4) to invite the Radnor fire and police departments to the parade?

   b. [/5] Of the 17 volunteers, only 7 have the necessary documentation to drive a Villanova van. How many ways are there to select three drivers for the vans that will take the students to the parade site?

   c. [/5] Suppose a group of 6 people are about to board the van. Only one of these has the documentation to drive. The van seats eight: a driver, a front passenger, a driver's side seatbelt, center seatbelt, passenger side seatbelt on the middle bench seat, and a driver's side seatbelt, center seatbelt, passenger side seatbelt on the rear bench seat. How many different seating arrangements are there? Note that the van must have a driver so how many ways can the five other persons sit?

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9. Recurrence Relations
    a. [___ / 5] For each of the following recurrences, state whether a closed form formula can be found by one of the methods we studied. If it can, say which method works. If it cannot, say which conditions are violated. In each case, say how many initial values are required.
        i. $a_n = a_{n-1} + 2n^2 - 4n + 37$

        ii. $a_n = a_{n-1} + 2^n + 5n - 3$

        iii. $a_n = a_{n-1}a_{n-2} + 8$

        iv. $a_n = 5a_{n-1} - 3a_{n-2} + 4$

        v. $a_n = a_{n-1} + 3a_{n-2} - 17a_{n-3}$

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b. [____ / 6] Solve the following recurrence relation using the method of characteristic equation. Show all your work carefully.
$a_n = a_{n-1} + 20a_{n-2}$ $a_0 = 3$ $a_1 = 2$

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10. Search trees
    a. [____/10] Create a binary search tree by inserting the following words in the order shown. You may ignore punctuation and upper vs lower case. (Sketch the tree on scrap paper first so that you can draw it carefully here, with enough room for all the branches.)

Binary Search, which Chapter ten discusses, is recursive in nature.

    b. [___/6] Show all comparisons done for each of the following searches in your tree
        i. Chapter

        ii. nature

        iii. two

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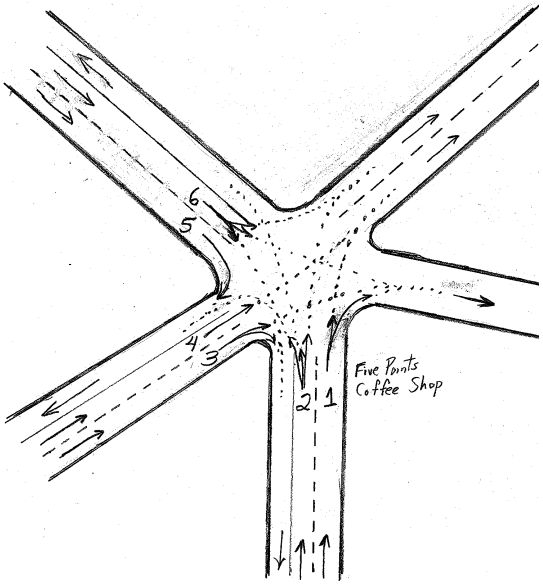
11. Eight miles from here, in Upper Darby, the Five Points Coffee Shop is at the intersection of five streets: Here is the Google Maps satellite photo and a diagram to show traffic flow:



Note that we have labeled the six lanes which enter this intersection.

a. [____ /10] Create a graph (as we did in class) to represent the potential accidents at this intersection.

b. [____/6] For the graph G in part (a.),
   i. Find the chromatic number.

   ii. Why is this number relevant to the design of traffic signals?

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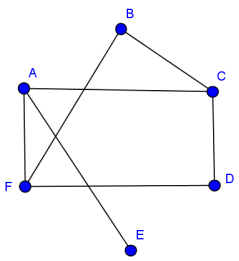
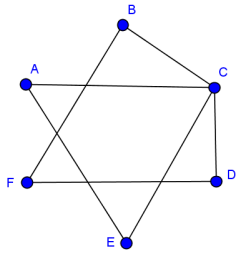
12. Here are two graphs.

Graph A



Graph B



a. [___/5] Identify which is bipartite. (Be sure to demonstrate that your selection is bipartite.)

b. [____/6] Find the chromatic number and chromatic index of this bipartite graph.

c. [_____/5] Find the degree sequence of this bipartite graph. Does the largest entry of the degree sequence give a lower bound for the chromatic number or for the chromatic index?

d. [___5] Find a perfect matching of this bipartite graph or indicate why it does not have one.

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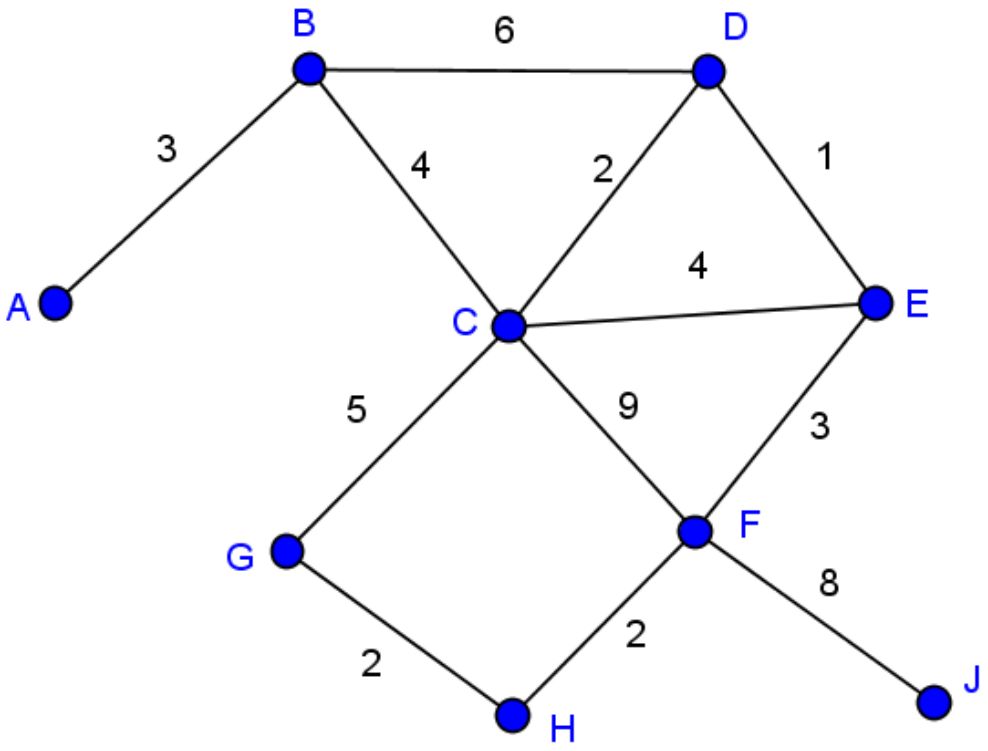
13. [____/10] [Dijkstra algorithm] Given this graph, use Dijkstra's algorithm to find the shortest path from node A to all other nodes. Show your work clearly. Just giving the answer will not be accepted. CLEARLY SHOW EACH STEP OF THE DIJKSTRA ALGORITHM IN ORDER. You will not need to use all of these copies.

If you do not wish to show the steps on the graphs, exhibit your trace in a consistent way that *shows the chronology of when each vertex is 'tagged.'*

In case you are accustomed to using extra copies of the graph, some are included.



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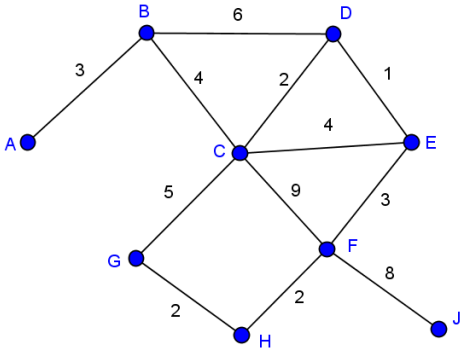
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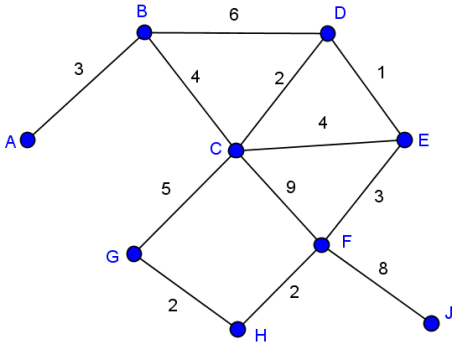
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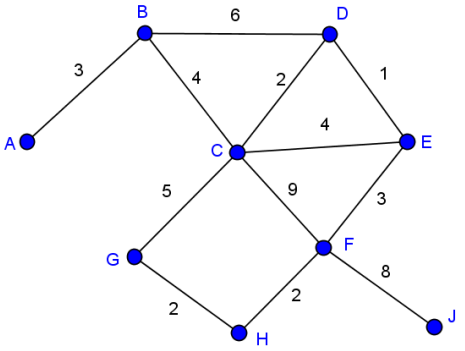
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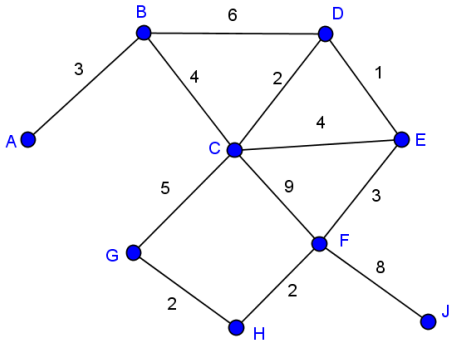
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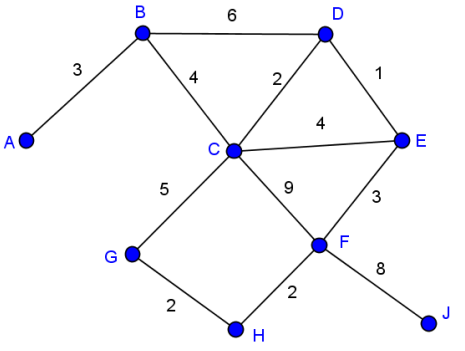
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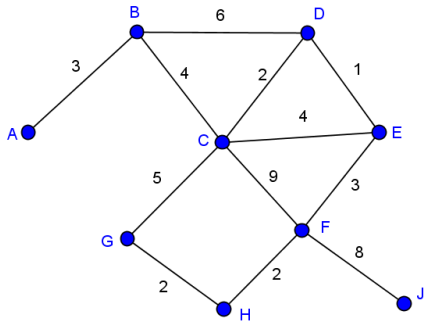
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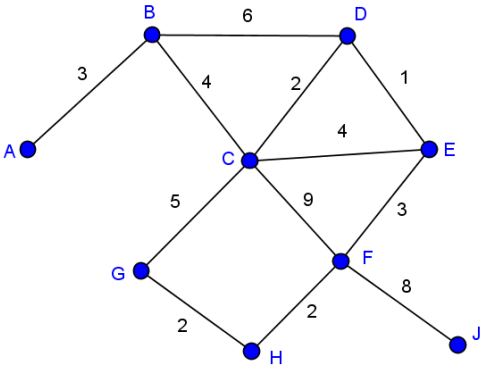
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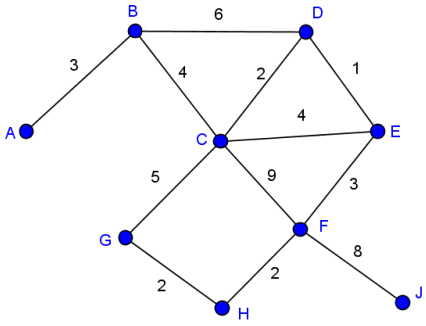
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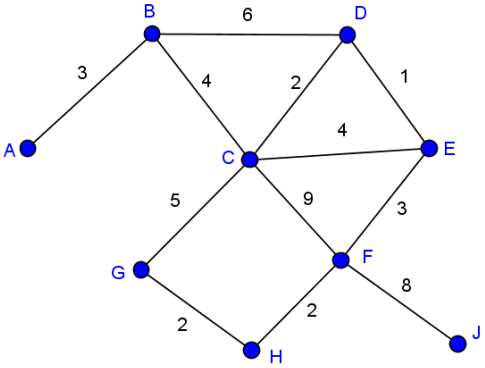
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