

CSC 1300 - Discrete Structures

Final Exam

May 6, 2015

Name: _____

Question	Value	Score
1	15	
2	25	
3	20	
4	15	
5	20	
6	20	
7	10	
8	10	
9	10	
10	20	
11	15	
12	20	
TOTAL	200	

No calculators or any reference except your one note card is permitted.

- Show your work carefully. Just writing an answer will not do. Show any assumptions; show the steps you took, and show how you came to your answer.
- If an answer involves a computation that would be very time consuming, you may leave it in a form that just needs final calculation except where you are explicitly asked to work a numeric answer.
- Please answer questions in the spaces provided. If you make a mistake or for some other reason need more space, please use the back of pages and clearly indicate where the answer can be found.

Good luck and best wishes for a great summer!

Alphabet chart and reference material

a	b	c	d	e	f	g	h	i	j	k	l	m	n	o	p	q	r	s	t	u	v	w	x	y	z

How many ways n labeled boxes?	at most one per box	any number per box	exactly one per box
k labeled (ordered) balls	A: $\binom{n}{k} k! = n(n-1)$ $\dots (n-k+1)$	E, F: $(k_j \text{ balls unordered within box})$ $\frac{k!}{k_1! k_2! \dots k_n!} = \binom{k}{k_1}$ $\binom{k-k_1}{k_2} \binom{k-k_1-k_2}{k_3}$ $\dots \binom{k_n+k_{n-1}}{k_{n-1}}$	_____
k unlabeled (unordered) balls	B: $\binom{n}{k}$	D, D': $\binom{k+n-1}{k} = \binom{k+n-1}{n-1}$ and $\binom{k-1}{n-1} = \binom{k-1}{k-n}$	_____
unlimited balls, k different labels (order matters)	_____	_____	C: k^n

1. [/15]
a. [/5] In a tournament with n teams, each team plays each other team exactly once. How many games are played in total. (Be sure to explain your answer.)

b. [/10] Given sets $A = \{w, x, y, z\}$ and $B = \{a, b\}$

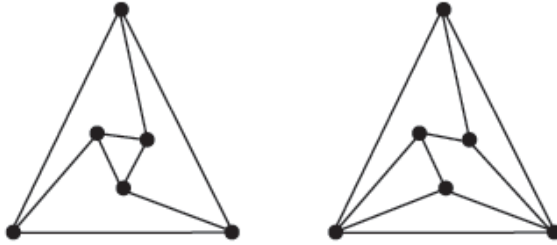
i. (4 pts) Show all functions from set A to set B

ii. (2pts) Which of those functions are 1 to 1? (Or explain why none exists.)

iii. (2pts) Which of those functions are onto? (Or explain why none exists.)

iv. (2pts) How many functions are there from a set of 25 elements to a set of 8 elements?

2. [/25]
 a. [/10] Are the two graphs isomorphic? If so, exhibit the isomorphism. If not, find a property that should be preserved by isomorphism for which the two graphs differ.



- b. [/10] Use a truth table to determine if $\neg(P \Rightarrow Q)$ is logically equivalent to $P \wedge \neg Q$

- c. [/5] Prove that if n is odd, then $n^2 + 5n - 3$ is also odd.

3.

[/20]

a. [/5] Write out in full:

$$\sum_{j=0}^7 \frac{2j-1}{2}$$

b. [/5] Using summation notation, write the following equation:

$$1 + 3 + 6 + 10 + \dots + \frac{n(n+1)}{2} = \frac{n(n+1)(n+2)}{6}$$

c. [/10] Prove that the equation above is true for all positive integers n by using induction.

4. [/15]

a. [/10] Decrypt the message:

Hkirmt szh urmzoob ziirevw zg Eroozmlez

which was encrypted using the Atbash Cipher. (Clearly show your final result – put a box around it or otherwise make it clear.)

b. [/5] Is $20 \equiv 12 \pmod{8}$ Yes or No:

Is $12 \equiv 20 \pmod{8}$ Yes or No:

Explain your answer:

5.

[/15]

a. [/10] How many different Sushi Samplers (three kinds of rolls) can be ordered from a 6-roll sushi menu (avocado, shrimp, yellowtail, dragon, kanpyo, eel)? Express your answer using choice notation and use Pascal's triangle identity to compute a numeric answer. (Note: for this question, you are required to give an actual number, a formula will not suffice.)

b. [/5] How many anagrams are there (including those that aren't dictionary words) of TELEPHONE?

6. [/20]
There are 28 sorority sisters that volunteered to go to an outreach program in West Philadelphia.

a. [/5] How many ways are there to select three students from this group of 28 sisters to be: 1) a group leader; 2) a photographer; and 3) a writer of an article for the Villanovan?

b. [/5] Of the 28 volunteers, only 10 have the necessary documentation to drive a Villanova van. How many ways are there to select four drivers for the vans that will take the students to the outreach program?

c. [/5] Let's assume that each van can hold at most 12 students, including the driver. What is the least number of vans needed to transport all the volunteers?

d. [/5] Suppose a group of 5 people are about to board the van. Only one of these is designated as a driver. How many ways are there to seat these 5 people in the van with another person in the front passenger seat with the driver, and the rest of them arranged in the 3-seater behind them?

7.

[/10]

a. [/5] The following are some statements about Fibonacci numbers. Mark each one as the statement is true for every value of $n \geq 0$, or the statement is false. Show your reasoning, but it is not necessary to supply a proof.

Recall that the Fibonacci sequence is defined by the recurrence $F_n = F_{n-1} + F_{n-2}$ with initial conditions $F_0 = 0$ and $F_1 = 1$.

- $F_{n+2} = F_n + F_{n+1}$
- $F_1 + F_3 + F_5 + \cdots + F_{2n-1} = F_{2n}$.
- $F_1 + F_3 + F_5 + \cdots + F_{2n-1} = F_{3n}$.
- F_{2n} is even.
- F_{3n} is even.

b. [/5] Consider binary numbers with n digits (e.g., 1010, 00101, 011). How many binary numbers of length n do not contain the substring 000? Denote this number by z_n ; find a relationship between z_n, z_{n-1} , and (we're not going to tell you) in order to form an appropriate recurrence relation. *DO NOT try to find a closed form, just write the recurrence and initial conditions and be sure to explain your reasoning.*

8. Find a closed form for $a_1 = 2, a_n = a_{n-1} + 2n$. Verify your solution for $n = 1, 2, 3, 4$ [/10]

Verify solution using this table

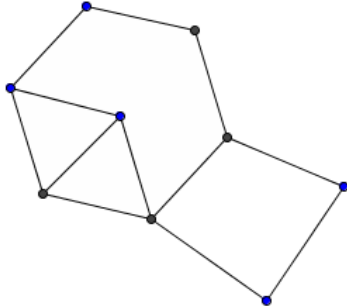
n	$a_n = a_{n-1} + 2n$	
1	2	
2		
3		
4		

Put your closed form in the highlighted box and show the value of your closed form for $n = 1, 2, 3, 4$ in the boxes below.

9.

[/10]

a. [/5] Find a perfect matching for the graph below, or explain why no perfect matching exists.



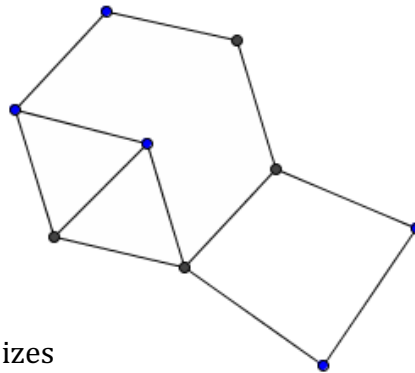
b. [/5] Make a binary search tree for this list of words. Place the first item shown at the root. Insert the other items **in the order in which they appear.**
if, it, is, not, too, much, of, an, inconvenience

10.

[/20]

a. (5 pts) Draw $K_{5,2}$ as a planar graph.

b.(15 pts) Given the following graph:



i. (5 pts) Label the faces with their sizes

ii. (5 pts) Illustrate **Euler's formula** for this graph. (Fill in the numbers for the terms in the formula.)

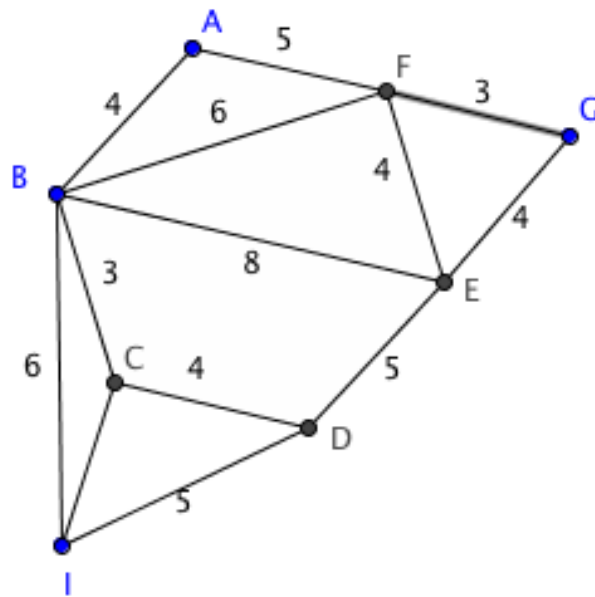
iii. (5 pts) Does this graph have an **Euler circuit** or an **Euler path**? Why or why not?

11.

[/15]

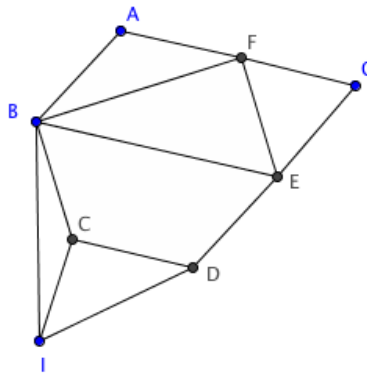
Given the following weighted graph, use Dijkstra's algorithm to find the shortest path from node A to **all other nodes**. Show your work clearly. Just giving the answer will not be accepted. YOU MUST CLEARLY SHOW THE STEPS OF THE DIJKSTRA ALGORITHM.

Show your results on the graph. Each entry must show the path length (total weight to that node) and the previous node on the shortest path from A.



12. Consider the graph:

[/20]



a. (5 pts) Since the graph is planar, what is the maximum number of colors needed to do a vertex coloring (i.e., how can you answer this without actually coloring the graph)? ____

b. (10 points) Using numbers to represent colors (unless you came with colored pencils), show the vertex coloring of the graph.

c. (5 pts) What is the chromatic number of the graph? _____