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# CSC 1300 Spring 2014
# Final Examination

150 minutes, 200 points. No devices with on/off switches, books, notes or other aids allowed. Villanova University academic integrity policy governs conduct of this exam.

For your use if/when needed:

| How many ways ... ... $n$ labeled boxes? | at most one per box | any number per box | exactly one per box |
|---|---|---|---|
| $k$ labeled (ordered) balls | A: $\binom{n}{k}k! = n(n-1)\ldots(n-k+1)$ | E, F: ($k_j$ balls unordered within box) $\frac{k!}{k_1!k_2!\ldots k_n!} = \binom{k}{k_1}$ $\binom{k-k_1}{k_2}\binom{k-k_1-k_2}{k_3}$ $\ldots\binom{k_n+k_{n-1}}{k_{n-1}}$ | ——— |
| $k$ unlabeled (unordered) balls | B: $\binom{n}{k}$ | D, D': $\binom{k+n-1}{k} = \binom{k+n-1}{n-1}$ and $\binom{k-1}{n-1} = \binom{k-1}{k-n}$ | ——— |
| unlimited balls, $k$ different labels (order matters) | ——— | ——— | C: $k^n$ |

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1. Is the product of two odd numbers even or odd? Prove it. (Direct proof)

2. Consider the sets

   A = {3, 8, 12, 15, 23, 75}

   B = {4, 10, 15, 20}

   C = {3, 4, 12, 15, 23}

   D = {3, 12, 23}

   a. List the elements of $A \cup C$

   b. List the elements of $A \cap D$

   c. List the elements of $A \backslash C$ $(aka\ A - C)$

   d. List the elements of $B \times D$

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3. Given sets A = {a, b, c} and B={4, 7}
    a. Show all functions from set A to set B

    b. Which of those functions are 1 to 1? Or explain why none exists

    c. Which of those functions are onto? Or explain why none exists

    d. How many functions are there from a set of 25 elements to a set of 8 elements?

4. Use DeMorgan's laws to find
    a. $\overline{A \cup B \cup C} =$

    b. $\overline{A \cap B \cup \bar{C}} =$

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5. Make a truth table to represent $(P \vee Q) \wedge R$

6. Draw a graph with degree sequence (1, 2, 2, 3, 4)

7. How many edges are in a star graph with $n$ vertices? Explain your answer.

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8. Use induction to show that
    a. $3\sum_{i=1}^{n} j(j-1) = n(n-1)(n-2)$

9. Which of the following are equivalence classes? Circle the ones that meet the criteria. For the ones that are not equivalence classes, say what condition(s) fail.
    a. All ducks of the same color

    b. $|a - b| < 8$

    c. Integers mod 5

    d. Integers $x, y$ such that $x^2 < y^2$

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10. Draw the first five rows of Pascal's triangle

11. What is the term containing $y^5$ in the expansion of $(3x + 4y)^7$

12. How many anagrams are there (including those that are not dictionary words) of COMPUTER?

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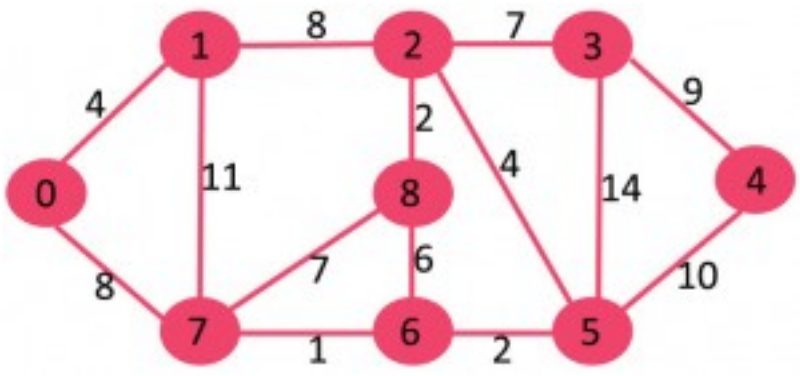
13. Given this graph, use Dijkstra's algorithm to find the shortest path from node O to **all other nodes**. Show your work clearly. Just giving the answer will not be accepted. YOU MUST CLEARLY SHOW THE STEPS OF THE DIJKSTRA ALGORITHM.




Show your results on the graph. Each entry should show the path length (total weight to that node) and the previous node on the shortest path from O.

Don – Showing the results on the graph might encourage following the algorithm steps?

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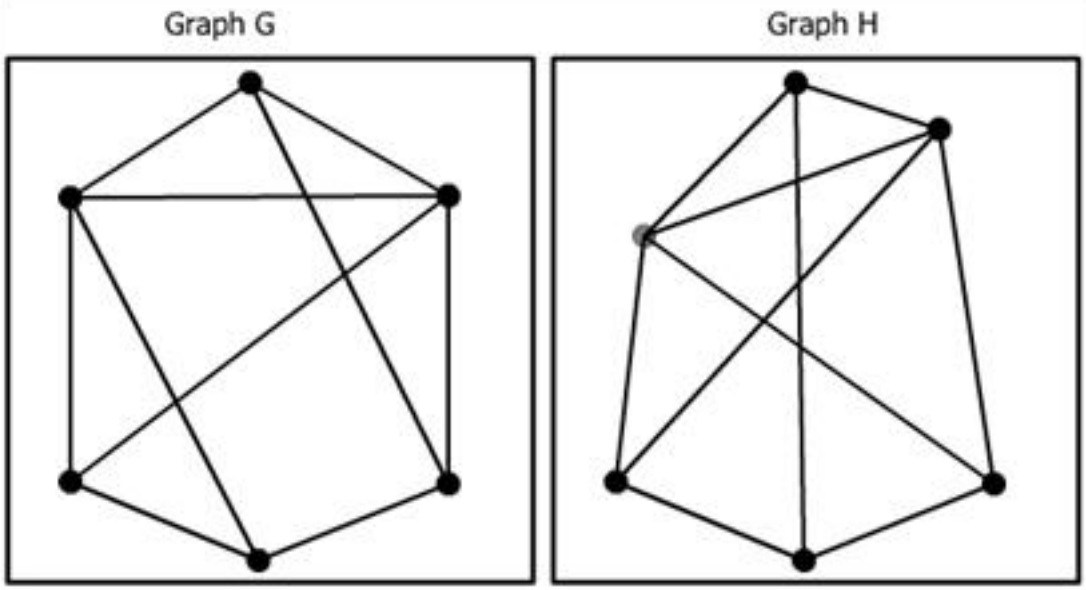
14. Are these graphs isomorphic? If so, label the vertices to show the equivalence. If not, say what requirement(s) are missing?



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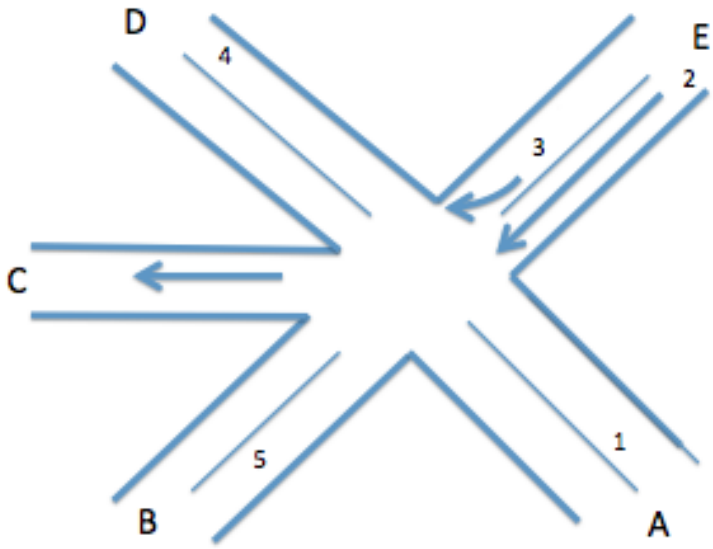
15. This diagram shows a traffic intersection. Rather than draw the whole graph that represents it (which would be very dense), answer some questions about the setup.




a. Draw in the traffic lanes, showing where potential conflicts arise. For example, traffic coming from part A of the graph could go to B or C or D. There are two lanes in part B. Show their full paths. Continue around the figure, showing all the places traffic can go.

b. Now, here is a graph representing some of the conflicts for traffic in this intersection. The vertices are labeled with the start and end graph area. For example, Vertex AE represents traffic coming from A and turning into E. Draw the necessary edges between pairs of vertices here.

c. In the five vertex subgraph you drew, what is the smallest number of traffic signals controlling this intersection. Explain your reasoning.