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CSC 1300 Spring 2017
Final Examination

150 minutes, 200 points. No devices with on/off switches, books, notes or other aids except your one note card allowed. Figure 7.1 is included for your reference.

Villanova University academic integrity policy governs conduct of this exam.

Question	Points Possible	Points Earned
1	20	
2	15	
3	8	
4	15	
5	16	
6	17	
7	10	
8	15	
9	8	
10	5	
11	10	
12	25	
13	15	
14	21	
Total	200	

How many ways ... ... n labeled boxes?	at most one per box	any number per box	exactly one per box
k labeled (ordered) balls	A: $\binom{n}{k}k! = n(n-1)\ldots(n-k+1)$	E, F: (k_j balls unordered within box) $\frac{k!}{k_1!k_2!\ldots k_n!} = \binom{k}{k_1}\binom{k-k_1}{k_2}\binom{k-k_1-k_2}{k_3}\ldots\binom{k_n+k_{n-1}}{k_{n-1}}$	———
k unlabeled (unordered) balls	B: $\binom{n}{k}$	D, D': $\binom{k+n-1}{k} = \binom{k+n-1}{n-1}$ and $\binom{k-1}{n-1} = \binom{k-1}{k-n}$	———
unlimited balls, k different labels (order matters)	———	———	C: k^n

Table 7.1. Solutions summary.

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1. Let A = { 1, 2, 3 } B = { a, {b,c} }

   a. [_____/6]:

      i. Find the set B x A by giving its elements

      ii. How many elements are in $\mathcal{P}(A)$ ("power set" of A)? _____

      iii. List them:

   b. [_____/3] How many functions *f* are there, *f:* A → B _____

   c. [_____/ 5] Enumerate all those functions (number them $f_1, f_2, ..$) by giving their action on each element of the domain. (Show the mapping from each element of the domain to something in the target space.)

   d. [_____/ 3] Which of those functions are 1-1? If none, explain why.

   e. [_____/ 3] Which of those functions are onto? If none, explain why.

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2. [_____/15] Induction: Prove

$$\sum_{j=0}^{n} 3^j = \frac{3^{n+1}-1}{2}$$

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3. [_____/8] Let $B_k = \{0, 1, \ldots k\}$ What is

   a. $\bigcup_{i=1}^{n} B_i, n \geq 1$ _____

   b. $\bigcap_{i=2}^{n} B_i, n \geq 2$ _____

4. [_____/15] Truth table: (Use as many rows and columns as you need.) Be sure to use the standard order for the P, Q, R entries. Show the truth table for the function

$$(P \Rightarrow Q) \wedge (\neg P \vee R)$$

P	Q	R					

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5. [_____/16] About modular arithmetic
   a. [2 points each] True or false? Explain your answer.
      i. $-4 \equiv 4 \ (mod\ 8)$

      ii. $-5 \equiv 5 \ (mod\ 8)$

      iii. $-8 \equiv 26 \ (mod\ 9)$

   b. [10 points] Given $a \equiv b \ (mod\ k)$ and $c \equiv d \ (mod\ k)$, prove $a - c \equiv b - d \ (mod\ k)$.

6. [_____/16] Graphs
   a. [6 points] Draw $K_{4,2}$ and show its adjacency matrix.

   b. [5 points] Is $K_{4,2}$ planar? Justify your answer.

   c. [6 points] Draw the complement of $K_{4,2}$ (relative to K_6)

7. [_____/10] Consider the relation defined on 3-bit strings as follows:
   $x \sim y$ iff x and y have the same number of ones.

   a. Prove that this is an equivalence relation.

   b. What are the equivalence classes of this relation? (List the elements of each equivalence class.)

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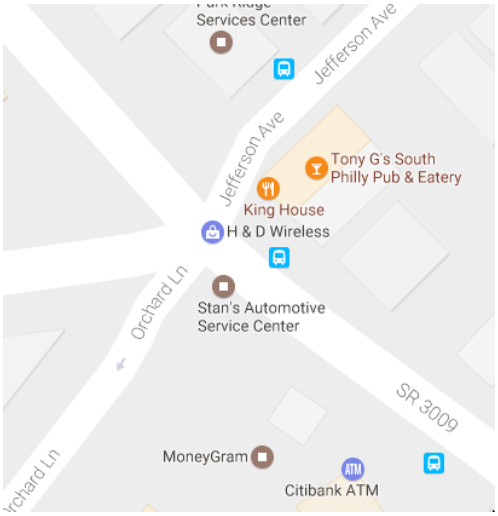
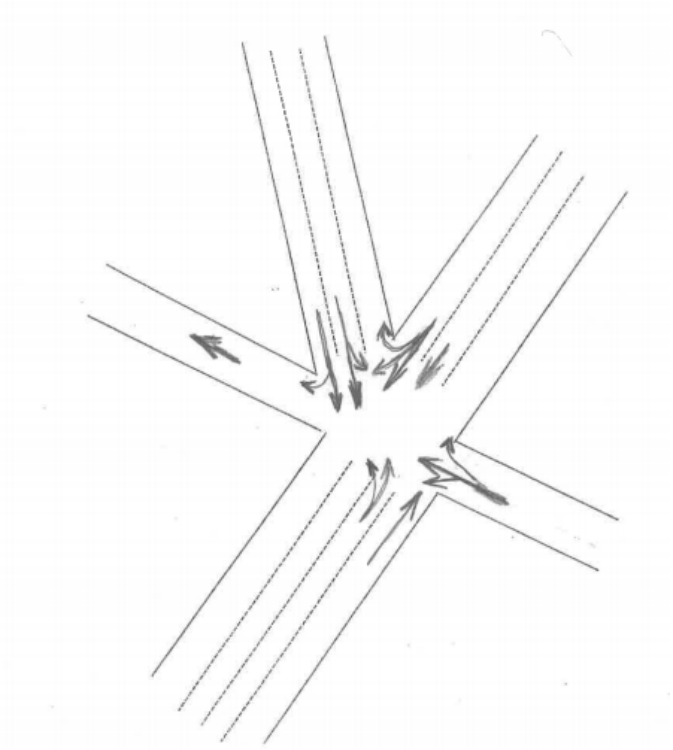
8. [_____/15] An intersection problem. We need to design the traffic signals for this intersection. Show the graph representation and the vertex coloring that corresponds to a good traffic flow. Carefully and clearly state any assumptions you make.





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9. [_____/8] What is the term that contains x^4 in the expansion of

$$(5x - 8y)^7$$

Show both the formula and the simplified value (ie, The end result has no binomial coefficient, no denominators. You do not have to multiply out the terms, though it is not hard to do so.).

10. [_____/5] Quickly calculate the value of this summation:

$$\sum_{i=0}^{23} \binom{23}{i} 8^i (-9)^{23-i}$$

11. [_____/10] It's feeding time at the zoo. There are 7 hungry chimps (clearly distinguishable from one another) and 26 identical bananas in the zoo keeper's basket. If each chimp is guaranteed at least 2 bananas, how many different ways are there to distribute the bananas to the chimps?

12. [_____/25] Using the graph in Figure 1, page 13 of this test,
    a. Give the degree sequence

    b. Verify the hand-shaking lemma

    c. Verify Euler's formula

    d. Find the chromatic number

    e. Find the chromatic index

13. [_____/15] Traversals, circuits**. Choice:** Do either part A or part B of this problem. **Only one part** will be graded. If you attempt both and do not indicate which one is to be graded, part A will be graded.

    Circle your choice: A or B

A: Dijkstra algorithm. Given the graph in Figure 1, use Dijkstra's algorithm to find the shortest path from node A to all other nodes. Show your work clearly. Just giving the answer will not earn points. Clearly show each step in order. You can do that by listing the edges included at each step. Extra copies of the graph are provided. Refer to edges by their end points in alphabetical order (AB, not BA). At each step, if you have a tie and have to make a choice, choose in alphabetical order. You may show the table of candidates at each step and show the labels as they are added to the graph.

OR

B: Circuits; Use the graph of Figure 1, Page 13 of the test.

a. Does this have a Hamilton circuit? If so, show it. If not, explain why.

b. Does this have an Euler circuit? If so, show it. If not, explain why.

14. [_____/21 (3 points each)] Let's investigate a simple, connected graph G, which has 19 vertices.

a. What is the minimum number of edges that it can have?

b. What is the maximum number of edges that it can have?

c. How many edges does a spanning tree for G have?

d. Can G have an odd number of vertices with odd degree? Why or why not?

e. Under what conditions does G have an Euler cycle?

f. What is $\Delta(G)$ (the maximum possible degree of a vertex in G)?

g. Let's assume G has 32 edges. If G is planar, how many faces does it have?

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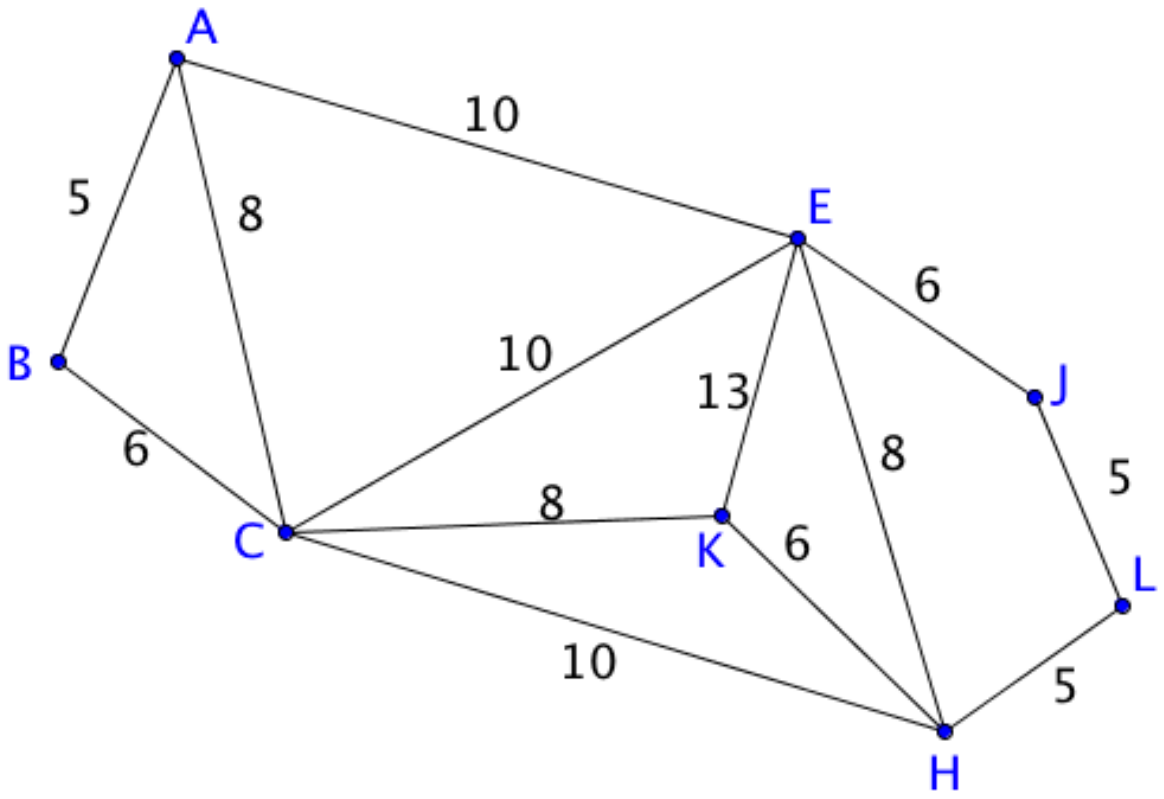
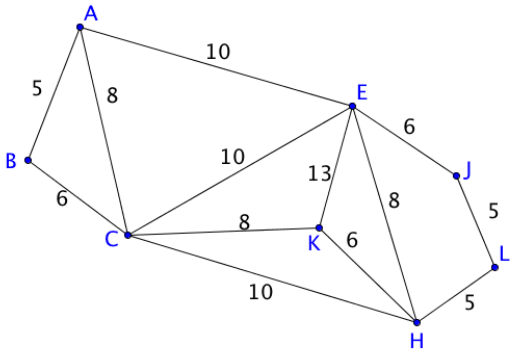
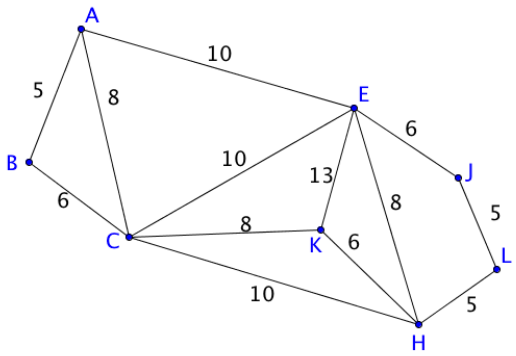


Figure 1 -- Graph for multiple questions

Extra copies of the graph to use as needed.





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