VILLANOVA UNIVERSITY
Department of Computing Sciences
Monday, February 2, 2015
CSC 1300 – 001

Exam 1

**Show your work carefully. Just writing an answer will not do. Show any assumptions;
show the steps you took, and show how you came to your answer**.

I. (20 points – 2 points each) Terminology
   For each of the following, write a brief but complete explanation of its
   meaning

   a. Handshaking lemma

   b. Contrapositive of $P \Rightarrow Q$

   c. Each of the following sets:
      i.   N

      ii. Z

      iii. W

   d. Each of the following graphs or graph components. Give the name for parts i,
      ii, iii; explain what it means for all of them.
      i. P_n

      ii. W_n

      iii. $K_{n,m}$

      iv. Leaf

      v.   Tree

II. (10 points. 5 points each)

a. Prove that the sum of two even numbers n_1 and n_2 is also even.

b. Class teams are forming by drawing chocolates from a bag.
   i.   If there are seven kinds of chocolate, how many candies must be drawn to be sure that one team has at least two members?

   ii. How many candies must be drawn to be sure that there is a team with at least 4 members?

III. (10 points) Product and Sum Principles
   Your favorite ice cream shop serves 8 types of ice cream: chocolate, vanilla, strawberry, mocha, coffee, mint chocolate chip, Moose tracks, and peanut butter swirl. They also have 3 kinds of sorbet: orange, lemon, and pineapple. They have 3 types of cones (waffle, sugar, cake) or you can have the ice cream, *but not sorbet,* in a dish.

   a. How many possible ways are there to enjoy a scoop of ice cream (not sorbet) at this shop?

   b. How many ways are there to have a scoop of sorbet?

   c. How many total ways are there to have an iced treat at the shop?

IV. (10 Points) Sets and Logic

    a. (2 points) List the elements of $\{z \in Z \mid -5 \leq z \leq 8\}$

    b. (5 points) List the elements of $\mathcal{P}(\{3, 5, \#, \{a,b\}\})$

        i.   How many elements are there? _____

        ii.   List the elements:

    c. (3 points) What is $\overline{A \cap B}$ =

        (Show the equivalent expression without the bar over the whole expression)

V. (5 points) Is the statement, "*If today is July 1, then it will snow tomorrow,*" true or false. Assume today's date is February 3. (Explain your answer)

VI. (5 points) Show the negation of the statement: Every child loves snow. (Write it with logic notation, then show the negative in logic notation, then convert to good English.)

VII.   (10 points) Truth Table
        Use a truth table to determine if the two expressions below are equivalent:

$(P \land Q \land \neg R) \lor (P \land \neg Q \land \neg R)$ $P \land \neg R$

VIII.   Graphs (5 points each = 20 points)
a. If G has *n* vertices and *e* edges, how many edges are there in $\bar{G}$? (Show your work)

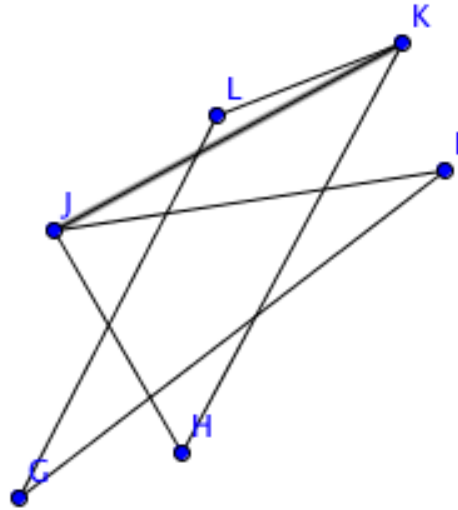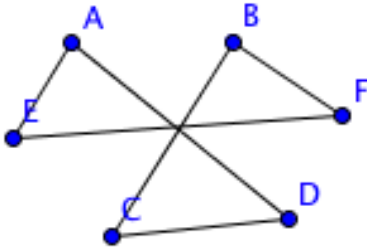b. Draw $\overline{W_6}$

c. Draw a graph with degree sequence (1,2,2,2,3)

d. Show that this graph is bipartite.
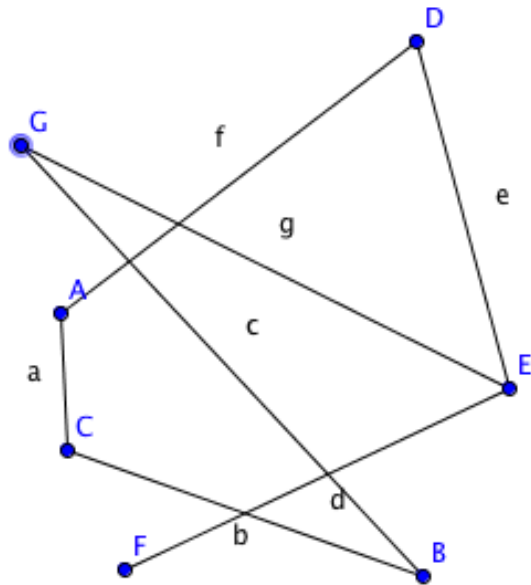
IX. (10 points) Isomorphism. If they are isomorphic, show the mapping. If they are not, explain how you came to that conclusion.

a. Are these graphs isomorphic?

b. Is either of them isomorphic to C_n for some n? (What n, if any)