

Name: _____

CSC 1300 Spring 2013
Exam 1 of 3

100 points. No devices with on/off switches, books, notes or other aids allowed. Villanova University academic integrity policy governs conduct of this exam.

Credit will be given only for legible work, clearly associated with a problem solution. Page backs are for scrap work and will not be considered part of the answers.

Some term or notation helps:

For sets, $A \setminus B$ is also written as $A - B$, $|A|$ is the size (cardinality) of A

A function is 1 to 1 if each element of the domain maps to a distinct element in the target.

A function is onto if there is a mapping to every element of the target.

1. Express clearly each of these. (You do not need to compute the actual number, but must show the result clearly. Also, you may not just put a number as the answer, but must show where the number comes from.)
 - a. How many 10-digit phone numbers are there? _____
 - b. How many 10-digit phone numbers are there that have no 9s?
 - c. How many 10-digit phone numbers are there that have at least one 9?
2. Prove that n^2 is odd iff n is odd.

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3. Suppose your deck of cards is standard: four suits, each with the values ace, king, queen, jack and 10 down to 2.
- How many cards would you need to draw to be assured that you get at least two of the same suit?
 - How many to be assured that you get at least two of the same value?

4. Let $A = \{-3, 15, 42\}$ $B = \{k \in \mathbb{N} \mid k / 3 \in \mathbb{N}\}$ $C = \{3, 6, 9, 12\}$

- How many subsets **of A** are there? _____
- Find $\mathcal{P}(A)$, the power set **of A** .

- How many elements are in the power set of D if D has m elements?

- Circle each of the statements below that are true.

$$A \subseteq B$$

$$B \cap A = \emptyset$$

$$C \subseteq B$$

$$C \cap B = C$$

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5. Given sets $A = \{a, b, c\}$ and $B = \{4, 7\}$
- Show all functions from set A to set B
 - Which of those functions are 1 to 1? Or explain why none exists
 - Which of those functions are onto? Or explain why none exists
 - How many functions are there from a set of 25 elements to a set of 8 elements?
6. One version of DeMorgan's law: $\neg(p \wedge q)$ is equivalent to $\neg p \vee \neg q$.
- Prove this using truth tables.
 - Write the equivalent law for sets, using A and B as the set names.

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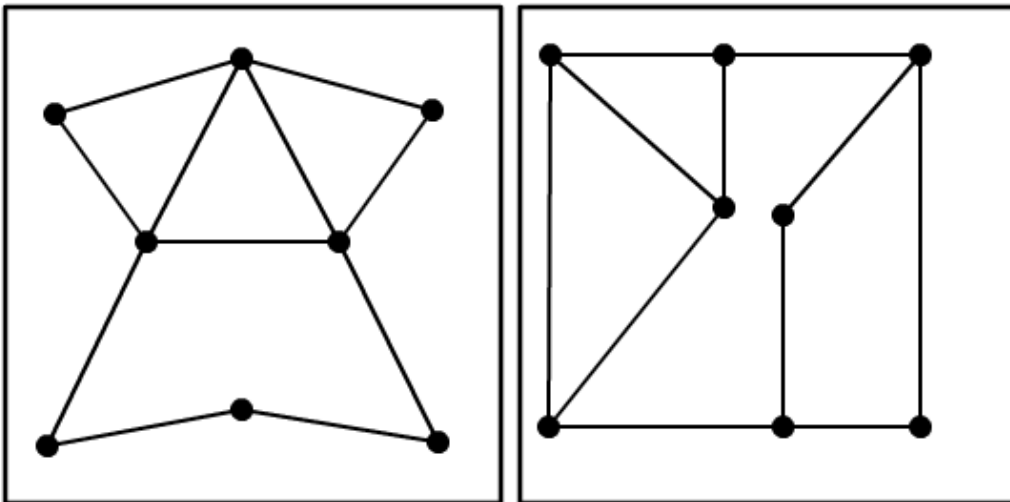
7. Let $A_k = \{0, 1, \dots, k\}$

a. What is $A_1, \underline{\hspace{2cm}}$ $A_2, \underline{\hspace{2cm}}$ $A_5, \underline{\hspace{2cm}}$

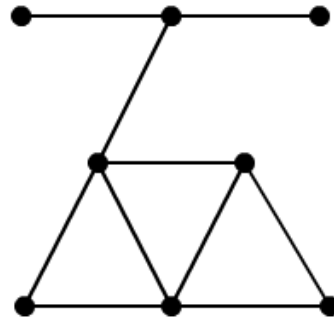
b. What is

$$\bigcup_{i=1}^n A_i$$

8. Are these graphs isomorphic? If no, point out why. If yes, label the second graph to show the corresponding vertices.



9. Given the graph shown here
a. What is the degree sequence?



- b. Show the adjacency matrix.

10. Consider the graph known as K_4
a. Sketch it

- b. Is it simple? Explain.

- c. Is it regular? Explain.

- d. Verify that the handshaking lemma holds.