Name: _____

Show your work carefully. Just writing an answer will not do. Show any assumptions; show the steps you took; and show how you came to your answer. And be sure to write legibly!

You may have use of one index card with whatever information you chose to put on it. No calculators or phones or any other information sources are permitted.

1. Show the adjacency matrix for this graph.
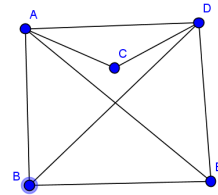
2. (5 pts) Evaluate the following summation. Please *write out all the terms first*. You should end up with a single fraction.

$$\sum_{i=0}^{4} \frac{i}{i+1}$$

3. Use induction to show that the following equality holds for all natural numbers n. Remember to write out each part of the induction proof carefully.

$$\sum_{j=0}^{n} (-1)^j \ j^2 = (-1)^n \ \frac{n(n+1)}{2}$$

4. For each one, state whether it is an equivalence relation or not. If it is not an equivalence relation, give all properties that fail (and tell why they fail):

    a. On the set of all persons, let u~v if and only if u and v speak a common language.

    b. On the set of all persons, let u~v if and only if u and v were born in the same country.

    c. On the set {a, b, c, d}, let u~v if and only if (u, v) is in the following set
    { (a,a), (b, b), (c,c), (d,d), (a, c), (b, c), (c, b), (c, a) }

5. Use the definition of congruence modulo N to prove that if $j \equiv k \pmod{N}$ then $7j \equiv 7k \pmod{N}$.

6. (5 pts) Encrypt the message below using shift cipher with shift 7. You will find it helpful to have the following table.

A	B	C	D	E	F	G	H	I	J	K	L	M
0	1	2	3	4	5	6	7	8	9	10	11	12
N	O	P	Q	R	S	T	U	V	W	X	Y	Z
13	14	15	16	17	18	19	20	21	22	23	24	25

Message: (You may choose not to use all the available lines. Do show all your steps clearly.)

E	N	C	R	Y	P	T	I	O	N	I	S	F	U	N

7. Consider the problem: In how many ways could you make an ice cream sundae, where you choose three different flavored scoops of ice cream from 8 possible flavors (the order of the scoops does not matter), and you choose four different toppings from 8 possible toppings?

   (a.)   Express the solution in choice notation: _____

   (b.)   Show Pascal's triangle down to the relevant row.

   (c.)   Indicate number of ways a sundae can be made: _____

8. Let $m \leq n$. Without writing out the math formulas, give a combinatorial scenario that illustrates what $\binom{n}{3}\binom{n}{m-3}$ might be counting?

9. In chess the rook can move along horizontal rows or vertical columns. How many ways can rooks be placed on an 8x8 chessboard so that each row and column has exactly one rook? What if the board is 10x10? Does this question ask for permutations or combinations?

10. Write down the term in the expansion of $(3x^2 + 6y^3)^5$ in which y^6 appears.

11. Use the binomial theorem (choose x and y appropriately) to show why the sum of each row of Pascal's triangle is a power of two, that is,

$$\sum_{j=0}^{n} \binom{n}{j} = 2^n$$