

VILLANOVA UNIVERSITY
Department of Computing Sciences
Tuesday, March 8, 2016

CSC 1300

Show your work carefully. Just writing an answer will not do. Show any assumptions; show the steps you took, and show how you came to your answer. Make sure your handwriting is clear and legible.

You may have the use of one index card with whatever information you chose to put on it. No calculators, phones, or any other information sources are permitted.

Exam 2 of 3

From Exam 1

1. $f(n) = \left\lfloor \frac{n}{3} \right\rfloor + 5$. Domain is \mathbb{N} , target space W

i. Find $f(4) =$

ii. Is $f(n)$ one-to-one? (Explain)

iii. Is f onto? (Explain)

Chapter 4

2. Write out in full. (Write out all the terms. You do not need to simplify or calculate the value.)

$$\sum_{j=0}^5 \frac{2j+1}{3}$$

3. Show by induction that every tree is bipartite.

4. Use induction to prove the following statement. Be sure to write out each step of the induction proof, properly labeled.

$$\sum_{i=1}^n i^3 = \frac{n^2(n+1)^2}{4}$$

Chapter 5

5. Show that

a. $<$ is not an equivalence relation

b. $=$ is an equivalence relation

c. We can express an equivalence relation on a set by showing ordered pairs that are equivalent. If the given pairs meet all the required conditions, then we have an equivalence relation.

Given the set $\{a, b, c, d\}$ and the relation \sim such that $x \sim y$ if and only if (x,y) is in the following set, is \sim an equivalence relation? Justify your answer by stating each requirement and illustrating that it is or is not met in this case.

$\{(a,a), (b,a), (a,c), (b,c), (b,b), (c,c), (a,b), (d,d), (c,b)\}$

Chapter 6

8. Pascal's triangle
- Write out the first 7 rows of the Pascal triangle.
 - Use the values in the triangle to show the number of ways to choose three flavors of ice cream from five choices. (First write the answer in choice notation, then find the answer in the triangle and circle it.)

9. Write $\binom{n}{0} - \binom{n}{1} + \binom{n}{2} - \binom{n}{3} + \binom{n}{4} \dots \pm \binom{n}{n}$ in summation notation. Then evaluate it, somehow. Explain what you are doing.

10. Quick binomial expansion questions

- Find the coefficient of the monomial containing d^4 in $(3c^2 - 5d)^8$
DO NOT DO A FULL EXPANSION OF THIS EXPRESSION. USE WHAT YOU HAVE LEARNED TO DO THIS EFFICIENTLY. You must show your work. Show the full coefficient, though you do not have to calculate the numeric value.

- Use the binomial theorem to compute the value of the following expression. Be sure to show your work. This should be quick.

$$\sum_{i=0}^m 8^i (-7)^{m-i} \binom{m}{i}$$