

Name: _____

VILLANOVA UNIVERSITY
Department of Computing Sciences
Thursday, March 2, 17

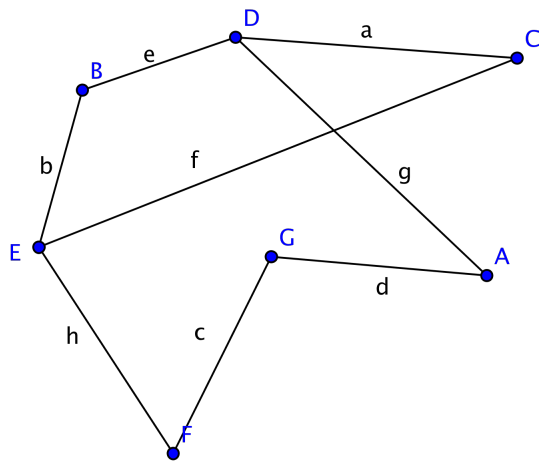
Exam 2 of 3

Show your work carefully. Just writing an answer will not do. Show any assumptions; show the steps you took; and show how you came to your answer.

You may have use of one index card with whatever information you chose to put on it. No calculators or phones or any other information sources are permitted.

From previous test: (Bipartite graph)

1. Is this graph bipartite? If so, show that clearly by redrawing the graph to show the required characteristics. If not, say what feature makes it not bipartite.



Chapter 4

2. Write as a summation: $1*2 + 2*3 + 3*4 + 4*5 + 5*6 + \dots + n*(n+1)$

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3. Evaluate the following summation. Be sure to write out the terms. You should end up with a number.

$$\sum_{i=1}^5 (-1)^{i+1} i^2$$

4. Use induction to show that, for a not equal to 1, the following equality holds. Remember to write out each part of the induction proof carefully.

$$\sum_{i=0}^n a^i = \frac{a^{n+1} - 1}{a - 1}$$

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Chapter 5

5. Given the set $\{1, 2, 3, 4, 5\}$

a. Explain why the set of pairs below **is** an equivalence relation:

$\{(1,1), (2,2), (3,3), (4,4), (5,5)\}$

(Show each required condition and show that it is met)

b. Does this set of pairs represent an equivalence relation? If so, say why. If not, say what condition is not met.

$\{(1,1), (1,2), (2,3), (2,2), (3,3), (4,4), (5,5), (3,2), (2,1)\}$

6. Prove, using only the definition of congruence modulo n that if $a \equiv b \pmod{n}$ and $c \equiv d \pmod{n}$, then $a + c \equiv b + d \pmod{n}$

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7. Use the **standard** Vigenère algorithm to decode the following message.

You will find it helpful to have the following table. Fill in the other row appropriately:

| | | | | | | | | | | | | | | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| a | b | c | d | e | f | g | h | i | j | k | l | m | n | o | p | q | r | s | t | u | v | w | x | y | z |
| | | | | | | | | | | | | | | | | | | | | | | | | | |

Keyword: TUESDAY

Show coded keyword: _____

Coded message: (You may choose not to use all the available lines. Do show your work clearly.)

| | | | | | | | | | | | | | | |
|-----------|-----------|-----------|-----------|----------|-----------|-----------|-----------|-----------|-----------|-----------|----------|-----------|----------|----------|
| 11 | 13 | 24 | 21 | 7 | 13 | 17 | 10 | 24 | 22 | 22 | 3 | 17 | 0 | 0 |
| | | | | | | | | | | | | | | |
| | | | | | | | | | | | | | | |
| | | | | | | | | | | | | | | |

Chapter 6

8. In how many ways can a committee of three faculty members and two students be formed from 8 faculty members and 7 students? (Express the solution using choice notation. Then solve for a value using Pascal's triangle.)

Expressed in choice notation: _____

Number of ways the committee can be formed: _____

Show (at least) the relevant rows of Pascal's triangle:

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9. What is the coefficient of x^3y^2 in the expansion of $(6x - 3y)^5$

10. Use the binomial theorem to compute the value of the following expression. Be sure to show your work. This should be very quick.

$$\sum_{i=0}^m 5^i (-4)^{m-i} \binom{m}{i}$$