

Name _____

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ShortExam 1 – Setting the Stage

40 minutes, 80 points. No devices with on/off switches, books, notes or other aids may be used. The University's academic integrity policy governs the conduct of this exam. Credit will be given only for legible work that is clearly associated with a problem statement. Page backs are for scrap work and and this work will not be considered part of the answer to a question. Read each question carefully. Ask the instructor if you do not understand the question or the notation.

Some terms and notation:

Set difference $A \setminus B$ is also written $A - B$.

Cardinality $|A|$ denotes the size (cardinality) of the set A .

One-to-one A function $f : A \rightarrow B$ is one to one if each element of the domain A maps to a distinct element in the target space (codomain) B .

Onto A function $f : A \rightarrow B$ is onto if each element of the target space (codomain) B is the image of some element in the domain A .

[10] 1. Let $S = \{a, b, c\}$.

(a) Find $\mathcal{P}(S)$, the power set of S .

(b) Find $|\mathcal{P}(S)|$.

(c) How many elements are in the power set of a set R if R contains m elements?

- [16] 2. Consider two functions f and g whose domains and codomains (target spaces in our book) are the natural numbers $\mathbf{N} = \{1, 2, 3, \dots\}$. Let $f(n) = 2n + 1$ and $g(n) = n^2$. Fill in the following table, placing a **T** in the cells indicating the corresponding proposition is true. In the cells where the corresponding proposition is false, place a counterexample. In the cells that ask for a computation, place your answer.

f is 1-1	
g is onto	
$\text{range}(f) =$	
$\text{range}(g) =$	

- [6] 3. Standard truck license plates in Pennsylvania consist of three letters followed by four numbers. However, the first letter is either a Y or a Z. How many possible truck license plates are there assuming that PennDOT uses a 22 letter alphabet?

- [8] 4. Let a and b be odd integers. Prove that $a^2 - b^2$ is an even integer.

[16] 5. Consider the sequence of sets $\{C_n\}$ where $C_n = \{j \in \mathbf{Z} \mid -n \leq j \leq n\}$ and $n = 0, 1, 2, \dots$

(a) Find C_1

(b) Find C_3

(c) Find $\bigcap_{n=0}^K C_n$ and its cardinality.

(d) Find $C_n \setminus C_{n-1}$ and its cardinality.

[8] 6. Draw a bipartite graph with 7 vertices and give its degree sequence.

- [10] 7. (*A tribute to a lazy hardware designer*) Let's define the *nand* operation on two propositions P and Q to give the value **False** if both P and Q are **True** and the value **True** otherwise. We write $P|Q$ to represent the *nand* operation.
- (a) Construct the truth table for $\neg(P \wedge Q)$.
- (b) What is the relation of the proposition in part (a) to the proposition $P|Q$?
- [6] 8. Are the two graphs below isomorphic? If yes, give the function that is the isomorphism. If no, give a reason that the graphs are different.